

Skill Check:

Must show work!!!

Solve $m - 8 \leq 14$.

a. $m \leq 6$

b. $m \geq 6$

c. $m \leq 22$

d. $m \geq 22$

In general, if x is any number and m, n are positive integers, then

$$x^m \cdot x^n = x^{m+n}$$

because

$$x^m \times x^n = \underbrace{(x \cdots x)}_{m \text{ times}} \times \underbrace{(x \cdots x)}_{n \text{ times}} = \underbrace{(x \cdots x)}_{m+n \text{ times}} = x^{m+n}.$$

In general, if x is nonzero and m, n are positive integers, then

$$\frac{x^m}{x^n} = x^{m-n}.$$

Zero Exponent

Words For any nonzero number a , $a^0 = 1$. The power 0^0 is undefined.

Numbers $4^0 = 1$ **Algebra** $a^0 = 1$, where $a \neq 0$

Negative Exponents

Words For any integer n and any nonzero number a , a^{-n} is the reciprocal of a^n .

Numbers $4^{-2} = \frac{1}{4^2}$ **Algebra** $a^{-n} = \frac{1}{a^n}$, where $a \neq 0$

EXAMPLE 1 Using Zero and Negative Exponents

Evaluate each expression.

a. $6 \cdot 7^0$

b. $(-2)^{-4}$

SOLUTION

a. $6 \cdot 7^0 = 1$

Definition of zero exponent

b. $(-2)^{-4} = \frac{1}{(-2)^4}$

Definition of negative exponent

$= \frac{1}{16}$

Simplify.

EXAMPLE 2 Simplifying an Expression

Simplify the expression $\frac{4x^0}{y^{-3}}$. Write your answer using only positive exponents.

SOLUTION

$$\frac{4x^0}{y^{-3}} = 4x^0y^3$$

Definition of negative exponent

$$= 4y^3$$

Definition of zero exponent

Evaluate the expression.

1. $(-9)^0$

2. 3^{-3}

3. $\frac{-5^0}{2^{-2}}$

4. Simplify the expression $\frac{3^{-2}x^{-5}}{y^0}$. Write your answer using only positive exponents.

In Exercises 5–12, evaluate the expression.

(See Example 1.)

5. $(-7)^0$

6. 4^0

7. 5^{-4}

8. $(-2)^{-5}$

9. $\frac{2^{-4}}{4^0}$

10. $\frac{5^{-1}}{-9^0}$

11. $\frac{-3^{-3}}{6^{-2}}$

12. $\frac{(-8)^{-2}}{3^{-4}}$

In Exercises 13–22, simplify the expression. Write your answer using only positive exponents. (See Example 2.)

13. x^{-7}

14. y^0

15. $9x^0y^{-3}$

16. $15c^{-8}d^0$

17. $\frac{2^{-2}m^{-3}}{n^0}$

18. $\frac{10^0r^{-11}s}{3^2}$

19. $\frac{4^{-3}d^0}{b^{-7}}$

20. $\frac{p^{-8}}{7^{-2}q^{-9}}$

21. $\frac{2^2y^{-6}}{8^{-1}z^0x^{-7}}$

22. $\frac{13x^{-5}y^0}{5^{-3}z^{-10}}$

Skill Check

$$16. 15c^{-8}d^0$$

Product of Powers Property

Let a be a real number, and let m and n be integers.

Words To multiply powers with the same base, add their exponents.

Numbers $4^6 \cdot 4^3 = 4^{6+3} = 4^9$ **Algebra** $a^m \cdot a^n = a^{m+n}$

Quotient of Powers Property

Let a be a nonzero real number, and let m and n be integers.

Words To divide powers with the same base, subtract their exponents.

Numbers $\frac{4^6}{4^3} = 4^{6-3} = 4^3$ **Algebra** $\frac{a^m}{a^n} = a^{m-n}$, where $a \neq 0$

Power of a Power Property

Let a be a real number, and let m and n be integers.

Words To find a power of a power, multiply the exponents.

Numbers $(4^6)^3 = 4^{6 \cdot 3} = 4^{18}$ **Algebra** $(a^m)^n = a^{mn}$

EXAMPLE 3 Using Properties of Exponents

Simplify each expression. Write your answer using only positive exponents.

a. $3^2 \cdot 3^6$

b. $\frac{(-4)^2}{(-4)^7}$

c. $(z^4)^{-3}$

SOLUTION

$$\begin{aligned} \text{a. } 3^2 \cdot 3^6 &= 3^{2+6} \\ &= 3^8 = 6561 \end{aligned}$$

Product of Powers Property

Simplify.

$$\begin{aligned} \text{b. } \frac{(-4)^2}{(-4)^7} &= (-4)^{2-7} \\ &= (-4)^{-5} \\ &= \frac{1}{(-4)^5} = -\frac{1}{1024} \end{aligned}$$

Quotient of Powers Property

Simplify.

Definition of negative exponent

$$\begin{aligned} \text{c. } (z^4)^{-3} &= z^{4 \cdot (-3)} \\ &= z^{-12} \\ &= \frac{1}{z^{12}} \end{aligned}$$

Power of a Power Property

Simplify.

Definition of negative exponent

Simplify the expression. Write your answer using only positive exponents.

5. $10^4 \cdot 10^{-6}$

6. $x^9 \cdot x^{-9}$

7. $\frac{-5^8}{-5^4}$

8. $\frac{y^6}{y^7}$

9. $(6^{-2})^{-1}$

10. $(w^{12})^5$

Power of a Product Property

Let a and b be real numbers, and let m be an integer.

Words To find a power of a product, find the power of each factor and multiply.

Numbers $(3 \cdot 2)^5 = 3^5 \cdot 2^5$ **Algebra** $(ab)^m = a^m b^m$

Power of a Quotient Property

Let a and b be real numbers with $b \neq 0$, and let m be an integer.

Words To find the power of a quotient, find the power of the numerator and the power of the denominator and divide.

Numbers $\left(\frac{3}{2}\right)^5 = \frac{3^5}{2^5}$ **Algebra** $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$, where $b \neq 0$

EXAMPLE 4 Using Properties of Exponents

Simplify each expression. Write your answer using only positive exponents.

a. $(-1.5y)^2$ b. $\left(\frac{a}{-10}\right)^3$ c. $\left(\frac{3d}{2}\right)^4$ d. $\left(\frac{2x}{3}\right)^{-5}$

SOLUTION

a. $(-1.5y)^2 = (-1.5)^2 \cdot y^2$ Power of a Product Property
 $= 2.25y^2$ Simplify.

b. $\left(\frac{a}{-10}\right)^3 = \frac{a^3}{(-10)^3}$ Power of a Quotient Property
 $= -\frac{a^3}{1000}$ Simplify.

c. $\left(\frac{3d}{2}\right)^4 = \frac{(3d)^4}{2^4}$ Power of a Quotient Property
 $= \frac{3^4 d^4}{2^4}$ Power of a Product Property
 $= \frac{81d^4}{16}$ Simplify.

d. $\left(\frac{2x}{3}\right)^{-5} = \frac{(2x)^{-5}}{3^{-5}}$ Power of a Quotient Property
 $= \frac{3^5}{(2x)^5}$ Definition of negative exponent
 $= \frac{3^5}{2^5 x^5}$ Power of a Product Property
 $= \frac{243}{32x^5}$ Simplify.

Simplify the expression. Write your answer using only positive exponents.

11. $(10y)^{-3}$ 12. $\left(-\frac{4}{n}\right)^5$ 13. $\left(\frac{1}{2k^2}\right)^5$ 14. $\left(\frac{6c}{7}\right)^{-2}$

Scientific Notation



EXAMPLE 6 Solving a Real-Life Problem

A jellyfish emits about 1.25×10^8 particles of light, or photons, in 6.25×10^{-4} second. How many photons does the jellyfish emit each second? Write your answer in scientific notation and in standard form.

SOLUTION

Divide to find the unit rate.

$$\frac{1.25 \times 10^8}{6.25 \times 10^{-4}}$$

← photons
← seconds

Write the rate.

$$= \frac{1.25}{6.25} \times \frac{10^8}{10^{-4}}$$

Rewrite.

$$= 0.2 \times 10^{12}$$

Simplify.

$$= 2 \times 10^{11}$$

Write in scientific notation.

► The jellyfish emits 2×10^{11} , or 200,000,000,000 photons per second.



Monitoring Progress



Help in English and Spanish at [BigIdeasMath.com](https://www.BigIdeasMath.com)

15. Write two expressions that represent the area of a base of the cylinder in Example 5.
16. It takes the Sun about 2.3×10^8 years to orbit the center of the Milky Way. It takes Pluto about 2.5×10^2 years to orbit the Sun. How many times does Pluto orbit the Sun while the Sun completes one orbit around the center of the Milky Way? Write your answer in scientific notation.