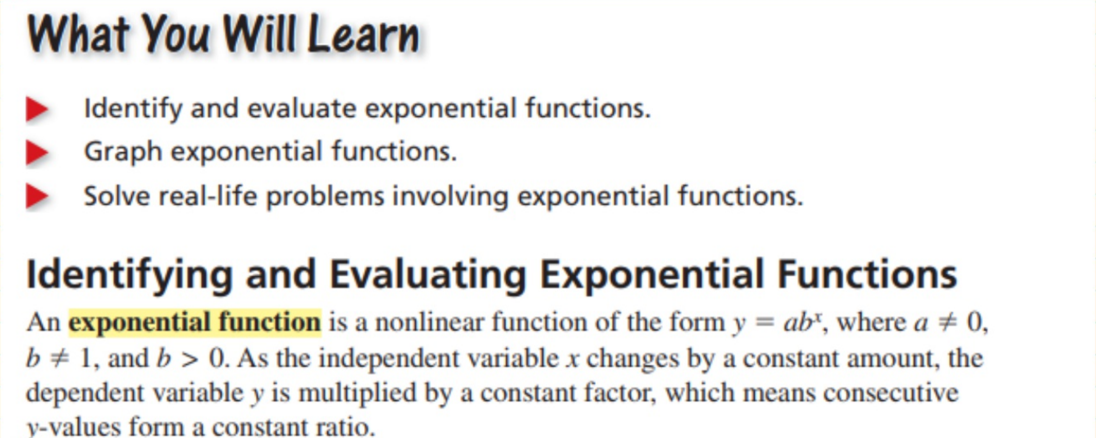
6-3 Exponential Functions

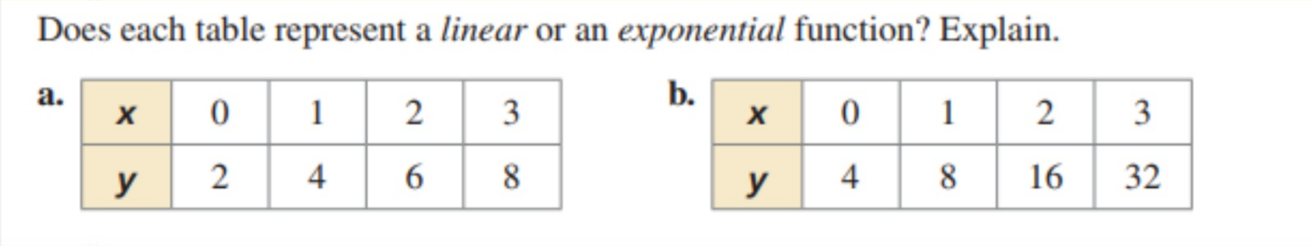


What You Will Learn

- Identify and evaluate exponential functions.
- Graph exponential functions.
- Solve real-life problems involving exponential functions.

Identifying and Evaluating Exponential Functions

An **exponential function** is a nonlinear function of the form $y = ab^x$, where $a \neq 0$, $b \neq 1$, and $b > 0$. As the independent variable x changes by a constant amount, the dependent variable y is multiplied by a constant factor, which means consecutive y-values form a constant ratio.



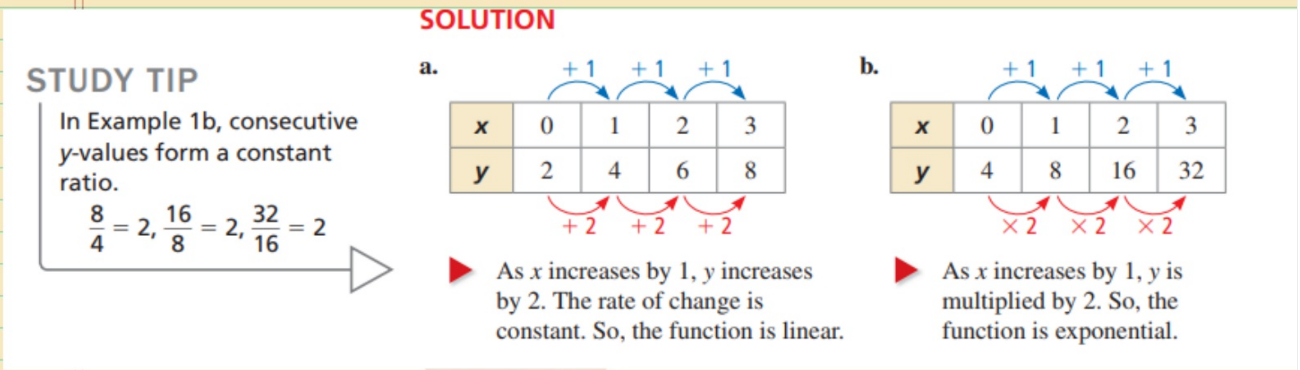
Does each table represent a *linear* or an *exponential* function? Explain.

a.

x	0	1	2	3
y	2	4	6	8

b.

x	0	1	2	3
y	4	8	16	32



STUDY TIP

In Example 1b, consecutive *y*-values form a constant ratio.

$\frac{8}{4} = 2, \frac{16}{8} = 2, \frac{32}{16} = 2$

SOLUTION

a.

+1 +1 +1

x	0	1	2	3
y	2	4	6	8

+2 +2 +2

▶ As x increases by 1, y increases by 2. The rate of change is constant. So, the function is linear.

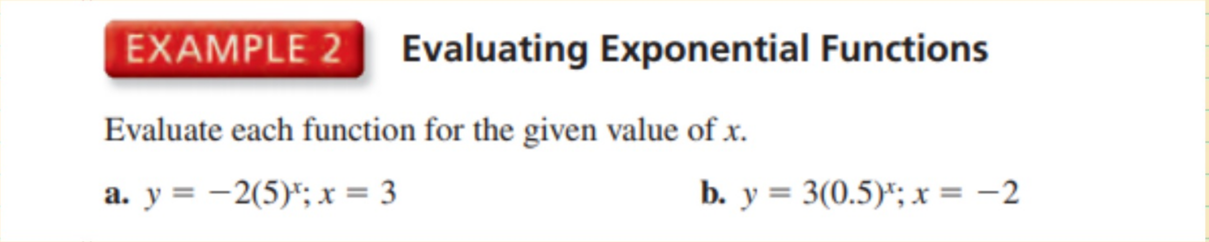
b.

+1 +1 +1

x	0	1	2	3
y	4	8	16	32

×2 ×2 ×2

▶ As x increases by 1, y is multiplied by 2. So, the function is exponential.

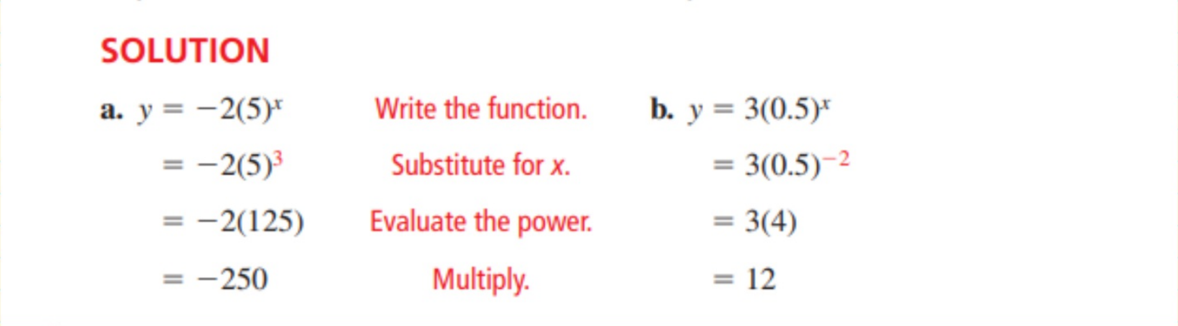


EXAMPLE 2 Evaluating Exponential Functions

Evaluate each function for the given value of x.

a. $y = -2(5)^x; x = 3$

b. $y = 3(0.5)^x; x = -2$



SOLUTION

a. $y = -2(5)^x$ Write the function.

$= -2(5)^3$ Substitute for *x*.

$= -2(125)$ Evaluate the power.

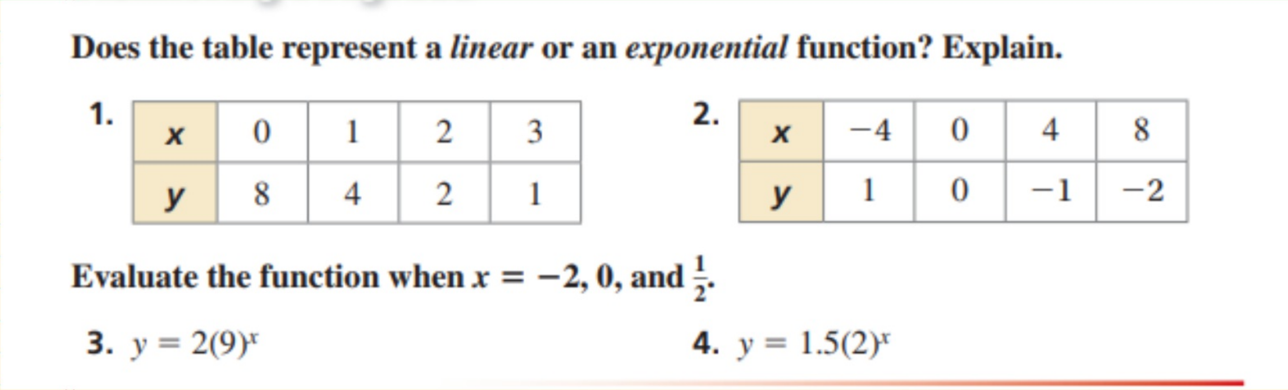
$= -250$ Multiply.

b. $y = 3(0.5)^x$

$= 3(0.5)^{-2}$

$= 3(4)$

$= 12$



Does the table represent a *linear* or an *exponential* function? Explain.

1.

x	0	1	2	3
y	8	4	2	1

2.

x	−4	0	4	8
y	1	0	−1	−2

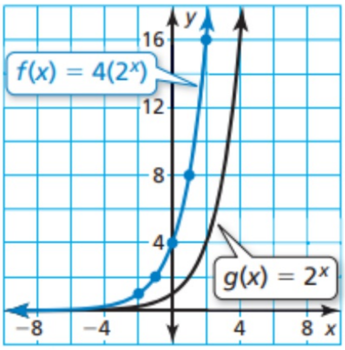
Evaluate the function when $x = -2, 0,$ and $\frac{1}{2}$.

3. $y = 2(9)^x$

4. $y = 1.5(2)^x$

EXAMPLE 3 Graphing $y = ab^x$ When $b > 1$

Graph $f(x) = 4(2)^x$. Compare the graph to the graph of the parent function. Describe the domain and range of f.



SOLUTION

Step 1 Make a table of values.

Step 2 Plot the ordered pairs.

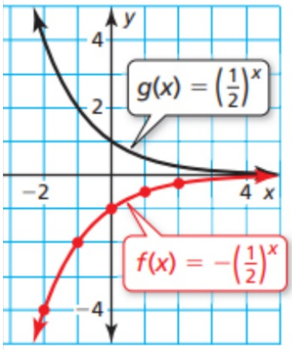
Step 3 Draw a smooth curve through the points.

x	−2	−1	0	1	2
***f*(*x*)**	1	2	4	8	16

▶ The parent function is $g(x) = 2^x$. The graph of f is a vertical stretch by a fa
4 of the graph of g. The y-intercept of the graph of f, 4, is above the y-inte
of the graph of g, 1. From the graph of f, you can see that the domain is al
numbers and the range is $y > 0$.

EXAMPLE 4 Graphing $y = ab^x$ When $0 < b < 1$

Graph $f(x) = -\left(\frac{1}{2}\right)^x$. Compare the graph to the graph of the parent function. Describe the domain and range of f.



SOLUTION

Step 1 Make a table of values.

Step 2 Plot the ordered pairs.

Step 3 Draw a smooth curve through the points.

x	-2	-1	0	1	2
$f(x)$	-4	-2	-1	$-\frac{1}{2}$	$-\frac{1}{4}$

The parent function is $g(x) = \left(\frac{1}{2}\right)^x$. The graph of f is a reflection in the x-axis of the graph of g. The y-intercept of the graph of f, -1, is below the y-intercept of the graph of g, 1. From the graph of f, you can see that the domain is all real numbers and the range is $y < 0$.

Graph the function. Compare the graph to the graph of the parent function. Describe the domain and range of *f*.

5. $f(x) = -2(4)^x$

6. $f(x) = 2\left(\frac{1}{4}\right)^x$

EXAMPLE 6 Comparing Exponential Functions

An exponential function g models a relationship in which the dependent variable is multiplied by 1.5 for every 1 unit the independent variable x increases. Graph g when $g(0) = 4$. Compare g and the function f from Example 3 over the interval $x = 0$ to $x = 2$.

SOLUTION

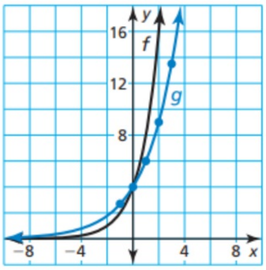
You know (0, 4) is on the graph of g. To find points to the right of (0, 4), multiply $g(x)$ by 1.5 for every 1 unit increase in x. To find points to the left of (0, 4), divide $g(x)$ by 1.5 for every 1 unit decrease in x.

Step 1 Make a table of values.

x	−1	0	1	2	3
$g(x)$	2.7	4	6	9	13.5

Step 2 Plot the ordered pairs.

Step 3 Draw a smooth curve through the points.



DY TIP

te that f is increasing
ter than g to the right
$x = 0$.

▶ Both functions have the same value when $x = 0$, but the value of f is greater than the value of g over the rest of the interval.

EXAMPLE 7 Modeling with Mathematics

The graph represents a bacterial population y after x days.

a. Write an exponential function that represents the population.

b. Find the population after 12 hours and after 5 days.

SOLUTION

1. **Understand the Problem** You have a graph of the population that shows some data points. You are asked to write an exponential function that represents the population and find the population after different amounts of time.
2. **Make a Plan** Use the graph to make a table of values. Use the table and the y-intercept to write an exponential function. Then evaluate the function to find the populations.
3. **Solve the Problem**

a. Use the graph to make a table of values.

+1 +1 +1 +1

x	0	1	2	3	4
y	3	12	48	192	768

×4 ×4 ×4 ×4

The y-intercept is 3. The y-values increase by a factor of 4 as x increases by 1.

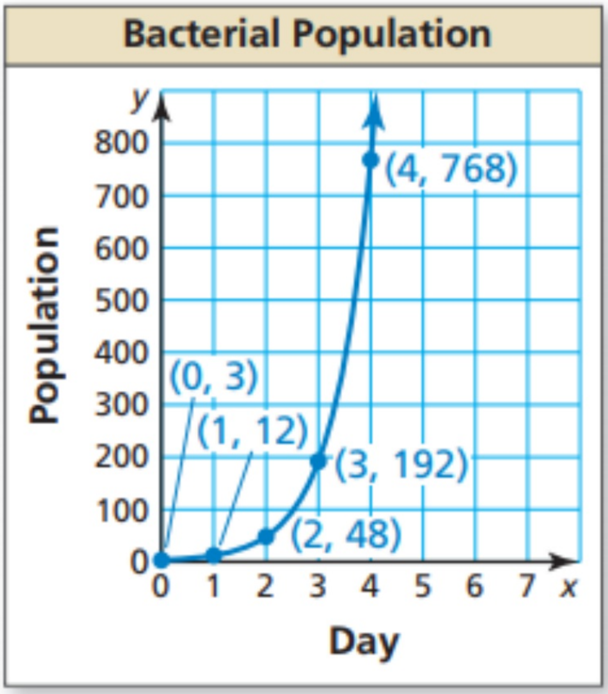
▶ So, the population can be modeled by $y = 3(4)^x$.

b. ***Population after 12 hours*** ***Population after 5 days***

hours $= \frac{1}{2}$ day

$y = 3(4)^x$	Write the function.	$y = 3(4)^x$
$= 3(4)^{1/2}$	Substitute for x.	$= 3(4)^5$
$= 3(2)$	Evaluate the power.	$= 3(1024)$
$= 6$	Multiply.	$= 3072$

▶ There are 6 bacteria after 12 hours and 3072 bacteria after 5 days.


Bacterial Population
y
800
700
600
500
400
300
200
100
0
Population
(0, 3)
(1, 12)
(2, 48)
(3, 192)
(4, 768)
0 1 2 3 4 5 6 7 x
Day