What You Will Learn

- Use and identify exponential growth and decay functions.
- Interpret and rewrite exponential growth and decay functions.
- Solve real-life problems involving exponential growth and decay.

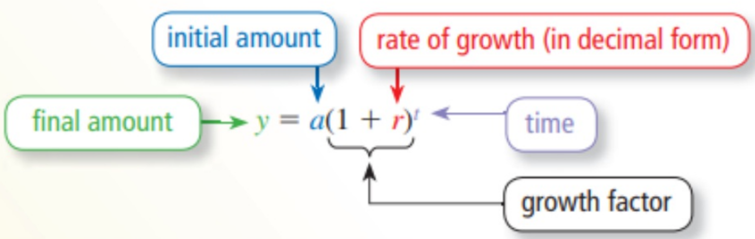
Exponential Growth and Decay Functions

Exponential growth occurs when a quantity increases by the same factor over equal intervals of time.

Core Concept

Exponential Growth Functions

A function of the form $y = a(1 + r)^t$, where $a > 0$ and $r > 0$, is an **exponential growth function**.



EXAMPLE 1 Using an Exponential Growth Function

The inaugural attendance of an annual music festival is 150,000. The attendance y increases by 8% each year.

a. Write an exponential growth function that represents the attendance after t years.

b. How many people will attend the festival in the fifth year? Round your answer to the nearest thousand.

SOLUTION

a. The initial amount is 150,000, and the rate of growth is 8%, or 0.08.

$y = a(1 + r)^t$ Write the exponential growth function.

$= 150{,}000(1 + 0.08)^t$ Substitute 150,000 for a and 0.08 for r.

$= 150{,}000(1.08)^t$ Add.

▶ The festival attendance can be represented by $y = 150{,}000(1.08)^t$.

b. The value $t = 4$ represents the fifth year because $t = 0$ represents the first year.

$y = 150{,}000(1.08)^t$ Write the exponential growth function.

$= 150{,}000(1.08)^4$ Substitute 4 for t.

$\approx 204{,}073$ Use a calculator.

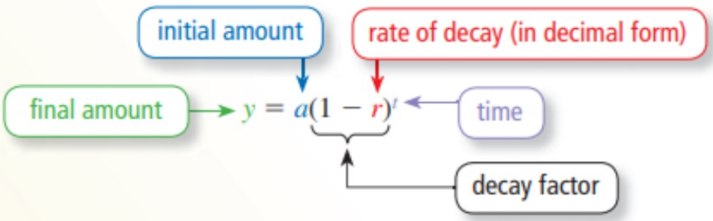
▶ About 204,000 people will attend the festival in the fifth year.

Exponential decay occurs when a quantity decreases by the same factor over equal intervals of time.

Core Concept

Exponential Decay Functions

A function of the form $y = a(1 - r)^t$, where $a > 0$ and $0 < r < 1$, is an **exponential decay function**.



For exponential growth, the value inside the parentheses is greater than 1 because r is added to 1. For exponential decay, the value inside the parentheses is less than 1 because r is subtracted from 1.

EXAMPLE 2 Identifying Exponential Growth and Decay

Determine whether each table represents an *exponential growth function*, an *exponential decay function*, or *neither*.

a.

x	*y*
0	270
1	90
2	30
3	10

b.

x	0	1	2	3
y	5	10	20	40

SOLUTION

a.

x	y
0	270
1	90
2	30
3	10

As x increases by 1, y is multiplied by $\frac{1}{3}$. So, the table represents an exponential decay function.

b.

x	0	1	2	3
y	5	10	20	40

As x increases by 1, y is multiplied by 2. So, the table represents an exponential growth function.

SOLUTION

a.

x	y
0	270
1	90
2	30
3	10

As x increases by 1, y is multiplied by $\frac{1}{3}$. So, the table represents an exponential decay function.

b.

x	0	1	2	3
y	5	10	20	40

As x increases by 1, y is multiplied by 2. So, the table represents an exponential growth function.

Determine whether the table represents an *exponential growth function*, an *exponential decay function*, or *neither*. Explain.

2.

x	0	1	2	3
y	64	16	4	1

3.

x	1	3	5	7
y	4	11	18	25

2. exponential decay; As x increases by 1, y is multiplied by $\frac{1}{4}$.

3. neither; As x increases by 2, y increases by 7.

Interpreting and Rewriting Exponential Functions

EXAMPLE 3 **Interpreting Exponential Functions**

Determine whether each function represents *exponential growth* or *exponential decay*. Identify the percent rate of change.

a. $y = 5(1.07)^t$

b. $f(t) = 0.2(0.98)^t$

SOLUTION

a. The function is of the form $y = a(1 + r)^t$, where $1 + r > 1$, so it represents exponential growth. Use the growth factor $1 + r$ to find the rate of growth.

$1 + r = 1.07$ Write an equation.

$r = 0.07$ Solve for r.

▶ So, the function represents exponential growth and the rate of growth is 7%.

b. The function is of the form $y = a(1 - r)^t$, where $1 - r < 1$, so it represents exponential decay. Use the decay factor $1 - r$ to find the rate of decay.

$1 - r = 0.98$ Write an equation.

$r = 0.02$ Solve for r.

▶ So, the function represents exponential decay and the rate of decay is 2%.

EXAMPLE 4 Rewriting Exponential Functions

Rewrite each function to determine whether it represents *exponential growth* or *exponential decay*.

a. $y = 100(0.96)^{t/4}$ **b.** $f(t) = (1.1)^{t-3}$

STUDY TIP

You can rewrite exponential expressions and functions using the properties of exponents. Changing the form of an exponential function can reveal important attributes of the function.

SOLUTION

a. $y = 100(0.96)^{t/4}$ Write the function.

$= 100(0.96^{1/4})^t$ Power of a Power Property

$\approx 100(0.99)^t$ Evaluate the power.

▶ So, the function represents exponential decay.

b. $f(t) = (1.1)^{t-3}$ Write the function.

$= \dfrac{(1.1)^t}{(1.1)^3}$ Quotient of Powers Property

$\approx 0.75(1.1)^t$ Evaluate the power and simplify.

▶ So, the function represents exponential growth.

Determine whether the function represents *exponential growth* or *exponential decay*. Identify the percent rate of change.

4. $y = 2(0.92)^t$

5. $f(t) = (1.2)^t$

Rewrite the function to determine whether it represents *exponential growth* or *exponential decay*.

6. $f(t) = 3(1.02)^{10t}$

7. $y = (0.95)^{t+2}$

4. exponential decay; 8%

5. exponential growth; 20%

6. $f(t) \approx 3(1.22)^t$; exponential growth

7. $y \approx 0.9(0.95)^t$; exponential decay

Solving Real-Life Problems

Exponential growth functions are used in real-life situations involving *compound interest*. Although interest earned is expressed as an *annual* rate, the interest is usually compounded more frequently than once per year. So, the formula $y = a(1 + r)^t$ must be modified for compound interest problems.

Core Concept

Compound Interest

Compound interest is the interest earned on the principal *and* on previously earned interest. The balance y of an account earning compound interest is

$$y = P\left(1 + \frac{r}{n}\right)^{nt}.$$

P = principal (initial amount)
r = annual interest rate (in decimal form)
t = time (in years)
n = number of times interest is compounded per year

STUDY TIP

For interest compounded yearly, you can substitute 1 for n in the formula to get $y = P(1 + r)^t$.

EXAMPLE 5 **Writing a Function**

You deposit \$100 in a savings account that earns 6% annual interest compounded monthly. Write a function that represents the balance after t years.

SOLUTION

$y = P\left(1 + \frac{r}{n}\right)^{nt}$ — Write the compound interest formula.

$= 100\left(1 + \frac{0.06}{12}\right)^{12t}$ — Substitute 100 for P, 0.06 for r, and 12 for n.

$= 100(1.005)^{12t}$ — Simplify.

EXAMPLE 6 Solving a Real-Life Problem

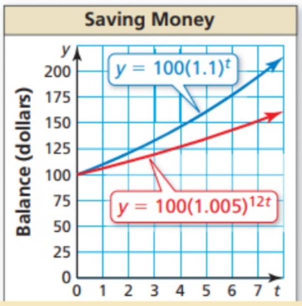
The table shows the balance of a money market account over time.

a. Write a function that represents the balance after t years.

b. Graph the functions from part (a) and from Example 5 in the same coordinate plane. Compare the account balances.

Year, t	Balance
0	\$100
1	\$110
2	\$121
3	\$133.10
4	\$146.41
5	\$161.05

SOLUTION



a. From the table, you know the initial balance is \$100, and it increases 10% each year. So, $P = 100$ and $r = 0.1$.

$y = P(1 + r)^t$ — Write the compound interest formula when $n = 1$.

$= 100(1 + 0.1)^t$ — Substitute 100 for P and 0.1 for r.

$= 100(1.1)^t$ — Add.

b. The money market account earns 10% interest each year, and the savings account earns 6% interest each year. So, the balance of the money market account increases faster.