

What You Will Learn

- ▶ Solve exponential equations with the same base.
- ▶ Solve exponential equations with unlike bases.
- ▶ Solve exponential equations by graphing.

Solving Exponential Equations with the Same Base

Exponential equations are equations in which variable expressions occur as exponents.

Core Concept

Property of Equality for Exponential Equations

Words Two powers with the *same positive base* b , where $b \neq 1$, are equal if and only if their exponents are equal.

Numbers If $2^x = 2^5$, then $x = 5$. If $x = 5$, then $2^x = 2^5$.

Algebra If $b > 0$ and $b \neq 1$, then $b^x = b^y$ if and only if $x = y$.

EXAMPLE 1 Solving Exponential Equations with the Same Base

Solve each equation.

a. $3^{x+1} = 3^5$

b. $6 = 6^{2x-3}$

c. $10^{3x} = 10^{2x+3}$

SOLUTION

a. $3^{x+1} = 3^5$

$$x + 1 = 5$$

$$\begin{array}{r} -1 \quad -1 \\ x = 4 \end{array}$$

b. $6 = 6^{2x-3}$

$$1 = 2x - 3$$

$$\begin{array}{r} +3 \quad +3 \\ 4 = 2x \end{array}$$

$$\frac{4}{2} = \frac{2x}{2}$$

$$2 = x$$

c. $10^{3x} = 10^{2x+3}$

$$3x = 2x + 3$$

$$\begin{array}{r} -2x \quad -2x \\ x = 3 \end{array}$$

Write the equation.

Equate the exponents.

Subtract 1 from each side.

Simplify.

Write the equation.

Equate the exponents.

Add 3 to each side.

Simplify.

Divide each side by 2.

Simplify.

Write the equation.

Equate the exponents.

Subtract 2x from each side.

Simplify.

Check

$$6 = 6^{2x-3}$$

$$6 \stackrel{?}{=} 6^{2(2)-3}$$

$$6 = 6 \quad \checkmark$$

Solve the equation. Check your solution.

1. $2^{2x} = 2^6$

2. $5^{2x} = 5^{x+1}$

3. $7^{3x+5} = 7^{x+1}$

1. $x = 3$

2. $x = 1$

3. $x = -2$

Solving Exponential Equations with Unlike Bases

To solve some exponential equations, you must first rewrite each side of the equation using the same base.

EXAMPLE 2 Solving Exponential Equations with Unlike Bases

Solve (a) $5^x = 125$, (b) $4^x = 2^{x-3}$, and (c) $9^{x+2} = 27^x$.

Check

$$4^x = 2^{x-3}$$

$$4^{-3} \stackrel{?}{=} 2^{-3-3}$$

$$\frac{1}{64} = \frac{1}{64} \quad \checkmark$$

Check

$$9^{x+2} = 27^x$$

$$9^4 \stackrel{?}{=} 27^4$$

$$531,441 = 531,441 \quad \checkmark$$

SOLUTION

a. $5^x = 125$

$5^x = 5^3$

$x = 3$

b. $4^x = 2^{x-3}$

$(2^2)^x = 2^{x-3}$

$2^{2x} = 2^{x-3}$

$2x = x - 3$

$x = -3$

c. $9^{x+2} = 27^x$

$(3^2)^{x+2} = (3^3)^x$

$3^{2x+4} = 3^{3x}$

$2x + 4 = 3x$

$4 = x$

Write the equation.

Rewrite 125 as 5^3 .

Equate the exponents.

Write the equation.

Rewrite 4 as 2^2 .

Power of a Power Property

Equate the exponents.

Solve for x .

Write the equation.

Rewrite 9 as 3^2 and 27 as 3^3 .

Power of a Power Property

Equate the exponents.

Solve for x .**EXAMPLE 3****Solving Exponential Equations When $0 < b < 1$** Solve (a) $\left(\frac{1}{2}\right)^x = 4$ and (b) $4^{x+1} = \frac{1}{64}$.

SOLUTION

a. $\left(\frac{1}{2}\right)^x = 4$
 $(2^{-1})^x = 2^2$
 $2^{-x} = 2^2$
 $-x = 2$
 $x = -2$

Write the equation.

Rewrite $\frac{1}{2}$ as 2^{-1} and 4 as 2^2 .

Power of a Power Property

Equate the exponents.

Solve for x .

b. $4^{x+1} = \frac{1}{64}$
 $4^{x+1} = \frac{1}{4^3}$
 $4^{x+1} = 4^{-3}$
 $x+1 = -3$
 $x = -4$

Write the equation.

Rewrite 64 as 4^3 .

Definition of negative exponent

Equate the exponents.

Solve for x .

Check

$$4^{x+1} = \frac{1}{64}$$
$$4^{-4+1} \stackrel{?}{=} \frac{1}{64}$$
$$\frac{1}{64} = \frac{1}{64} \quad \checkmark$$

Solve the equation. Check your solution.

4. $4^x = 256$

5. $9^{2x} = 3^{x-6}$

6. $4^{3x} = 8^{x+1}$

7. $\left(\frac{1}{3}\right)^{x-1} = 27$

4. $x = 4$

5. $x = -2$

6. $x = 1$

7. $x = -2$