Skill Check:

In Exercises 21–30, use the FOIL Method to find the product. *(See Example 3.)*

21. $(b + 3)(b + 7)$ **22.** $(w + 9)(w + 6)$

7-3 Special Products of Polynomials

1.) Use the square for a binomial pattern

2.) Use the sum and difference pattern

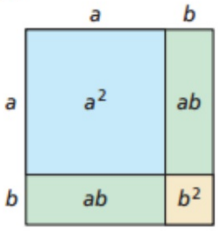
3.) Use special product pattern to solve real-life problems

Using the square of a Binomial Pattern

Using the Square of a Binomial Pattern

The diagram shows a square with a side length of $(a + b)$ units. You can see that the area of the square is

$$(a + b)^2 = a^2 + 2ab + b^2.$$



This is one version of a pattern called the square of a binomial. To find another version of this pattern, use algebra: replace b with $-b$.

$(a + (-b))^2 = a^2 + 2a(-b) + (-b)^2$ Replace b with $-b$ in the pattern above.

$(a - b)^2 = a^2 - 2ab + b^2$ Simplify.

Core Concept

Square of a Binomial Pattern

Algebra	Example
$(a + b)^2 = a^2 + 2ab + b^2$	$(x + 5)^2 = (x)^2 + 2(x)(5) + (5)^2$ $= x^2 + 10x + 25$
$(a - b)^2 = a^2 - 2ab + b^2$	$(2x - 3)^2 = (2x)^2 - 2(2x)(3) + (3)^2$ $= 4x^2 - 12x + 9$

Example 1: Using the square of a Binomial Pattern

Find each product.

a. $(3x + 4)^2$ **b.** $(5x - 2y)^2$

SOLUTION

a. $(3x + 4)^2 = (3x)^2 + 2(3x)(4) + 4^2$ Square of a binomial pattern

$= 9x^2 + 24x + 16$ Simplify.

▶ The product is $9x^2 + 24x + 16$.

b. $(5x - 2y)^2 = (5x)^2 - 2(5x)(2y) + (2y)^2$ Square of a binomial pattern

$= 25x^2 - 20xy + 4y^2$ Simplify.

▶ The product is $25x^2 - 20xy + 4y^2$.

Find the product.

1. $(x + 7)^2$ 2. $(7x - 3)^2$ 3. $(4x - y)^2$ 4. $(3m + n)^2$

Using the Sum and Difference Pattern

To find the product $(x + 2)(x - 2)$, you can multiply the two binomials using the FOIL Method.

$$(x + 2)(x - 2) = x^2 - 2x + 2x - 4 \qquad \text{FOIL Method}$$

$$= x^2 - 4 \qquad \text{Combine like terms.}$$

This suggests a pattern for the product of the sum and difference of two terms.

Core Concept

Sum and Difference Pattern

Algebra	Example
$(a + b)(a - b) = a^2 - b^2$	$(x + 3)(x - 3) = x^2 - 9$

Example 2: using the sum or difference method:

Find each product.

a. $(t + 5)(t - 5)$ **b.** $(3x + y)(3x - y)$

SOLUTION

a. $(t + 5)(t - 5) = t^2 - 5^2$ Sum and difference pattern

$= t^2 - 25$ Simplify.

▶ The product is $t^2 - 25$.

b. $(3x + y)(3x - y) = (3x)^2 - y^2$ Sum and difference pattern

$= 9x^2 - y^2$ Simplify.

▶ The product is $9x^2 - y^2$.

The special product patterns can help you use mental math to find certain products of numbers.

Example 3: Using special Product Patterns and Mental math

Use special product patterns to find the product $26 \cdot 34$.

SOLUTION

Notice that 26 is 4 less than 30, while 34 is 4 more than 30.

$$26 \cdot 34 = (30 - 4)(30 + 4)$$ Write as product of difference and sum.

$$= 30^2 - 4^2$$ Sum and difference pattern

$$= 900 - 16$$ Evaluate powers.

$$= 884$$ Simplify.

▶ The product is 884.

Find the product.

5. $(x + 10)(x - 10)$ **6.** $(2x + 1)(2x - 1)$ **7.** $(x + 3y)(x - 3y)$

8. Describe how to use special product patterns to find 21^2.

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5. $x^2 - 100$

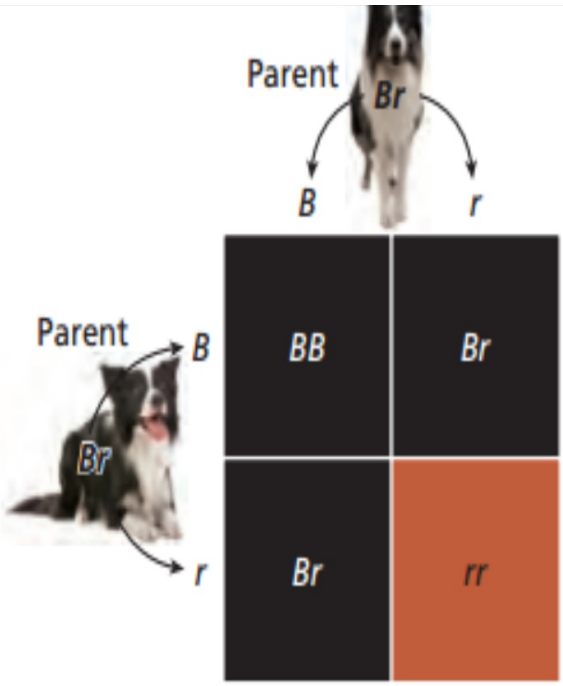
6. $4x^2 - 1$

7. $x^2 - 9y^2$

8. Rewrite 21^2 as $(20 + 1)^2$, then use the square of a binomial pattern $(20 + 1)^2 = 20^2 + 2 \cdot 20 \cdot 1 + 1^2 = 400 + 40 + 1 = 441$.

A combination of two genes determines the color of the dark patches of a border collie's coat. An offspring inherits one patch color gene from each parent. Each parent has two color genes, and the offspring has an equal chance of inheriting either one.

The gene B is for black patches, and the gene r is for red patches. Any gene combination with a B results in black patches. Suppose each parent has the same gene combination Br. The Punnett square shows the possible gene combinations of the offspring and the resulting patch colors.



a. What percent of the possible gene combinations result in black patches?

b. Show how you could use a polynomial to model the possible gene combinations.

SOLUTION

a. Notice that the Punnett square shows four possible gene combinations of the offspring. Of these combinations, three result in black patches.

▶ So, 75% of the possible gene combinations result in black patches.

b. Model the gene from each parent with $0.5B + 0.5r$. There is an equal chance that the offspring inherits a black or a red gene from each parent.

You can model the possible gene combinations of the offspring with $(0.5B + 0.5r)^2$. Notice that this product also represents the area of the Punnett square.

Expand the product to find the possible patch colors of the offspring.

$$(0.5B + 0.5r)^2 = (0.5B)^2 + 2(0.5B)(0.5r) + (0.5r)^2$$
$$= 0.25B^2 + 0.5Br + 0.25r^2$$

Consider the coefficients in the polynomial.

$$0.25B^2 + 0.5Br + 0.25r^2$$

25% BB, black patches | 50% Br, black patches | 25% rr, red patches

The coefficients show that 25% + 50% = 75% of the possible gene combinations result in black patches.