

Skill check:

a. $(a + 8)(a - 8)$

7-4 solving polynomials equations in standard form:

A polynomial is in **factored form** when it is written as a product of factors.

Standard form

$$x^2 + 2x$$

$$x^2 + 5x - 24$$

Factored form

$$x(x + 2)$$

$$(x - 3)(x + 8)$$

Zero property:

When one side of an equation is a polynomial in factored form and the other side is 0, use the **Zero-Product Property** to solve the polynomial equation. The solutions of a polynomial equation are also called **roots**.

Core Concept

Zero-Product Property

Words If the product of two real numbers is 0, then at least one of the numbers is 0.

Algebra If a and b are real numbers and $ab = 0$, then $a = 0$ or $b = 0$.

EXAMPLE 1 Solving Polynomial Equations

Solve each equation.

a. $2x(x - 4) = 0$

b. $(x - 3)(x - 9) = 0$

SOLUTION

a. $2x(x - 4) = 0$

$$2x = 0 \quad \text{or} \quad x - 4 = 0$$

$$x = 0 \quad \text{or} \quad x = 4$$

▶ The roots are $x = 0$ and $x = 4$.

b. $(x - 3)(x - 9) = 0$

$$x - 3 = 0 \quad \text{or} \quad x - 9 = 0$$

$$x = 3 \quad \text{or} \quad x = 9$$

▶ The roots are $x = 3$ and $x = 9$.

Write equation.

Zero-Product Property

Solve for x .

Write equation.

Zero-Product Property

Solve for x .

Solve the equation. Check your solutions.

1. $x(x - 1) = 0$

2. $3t(t + 2) = 0$

3. $(z - 4)(z - 6) = 0$

MONITORING PROGRESS ANSWERS

1. $x = 0, x = 1$
2. $t = 0, t = -2$
3. $z = 4, z = 6$

When two or more roots of an equation are the same number, the equation has **repeated roots**.

EXAMPLE 2 Solving Polynomial Equations

Solve each equation.

a. $(2x + 7)(2x - 7) = 0$ **b.** $(x - 1)^2 = 0$ **c.** $(x + 1)(x - 3)(x - 2) = 0$

SOLUTION

a. $(2x + 7)(2x - 7) = 0$

$$2x + 7 = 0 \quad \text{or} \quad 2x - 7 = 0$$

$$x = -\frac{7}{2} \quad \text{or} \quad x = \frac{7}{2}$$

▶ The roots are $x = -\frac{7}{2}$ and $x = \frac{7}{2}$.

b. $(x - 1)^2 = 0$

$$(x - 1)(x - 1) = 0$$

$$x - 1 = 0 \quad \text{or} \quad x - 1 = 0$$

$$x = 1 \quad \text{or} \quad x = 1$$

▶ The equation has repeated roots of $x = 1$.

c. $(x + 1)(x - 3)(x - 2) = 0$

$$x + 1 = 0 \quad \text{or} \quad x - 3 = 0 \quad \text{or} \quad x - 2 = 0$$

$$x = -1 \quad \text{or} \quad x = 3 \quad \text{or} \quad x = 2$$

▶ The roots are $x = -1$, $x = 3$, and $x = 2$.

Write equation.

Zero-Product Property

Solve for x .

Write equation.

Expand equation.

Zero-Product Property

Solve for x .

Write equation.

Zero-Product Property

Solve for x .

Solve the equation. Check your solutions.

4. $(3s + 5)(5s + 8) = 0$ 5. $(b + 7)^2 = 0$ 6. $(d - 2)(d + 6)(d + 8) = 0$

MONITORING PROGRESS ANSWERS

4. $s = -\frac{5}{3}, s = -\frac{8}{5}$

5. $b = -7$

6. $d = 2, d = -6, d = -8$

7. $8y(y - 3)$

EXAMPLE 3 Finding the Greatest Common Monomial Factor

Factor out the greatest common monomial factor from $4x^4 + 24x^3$.

SOLUTION

The GCF of 4 and 24 is 4. The GCF of x^4 and x^3 is x^3 . So, the greatest common monomial factor of the terms is $4x^3$.

► So, $4x^4 + 24x^3 = 4x^3(x + 6)$.

EXAMPLE 4 Solving Equations by Factoring

Solve (a) $2x^2 + 8x = 0$ and (b) $6n^2 = 15n$.

SOLUTION

a. $2x^2 + 8x = 0$

$$2x(x + 4) = 0$$

$$2x = 0 \quad \text{or} \quad x + 4 = 0$$

$$x = 0 \quad \text{or} \quad x = -4$$

▶ The roots are $x = 0$ and $x = -4$.

b. $6n^2 = 15n$

$$6n^2 - 15n = 0$$

$$3n(2n - 5) = 0$$

$$3n = 0 \quad \text{or} \quad 2n - 5 = 0$$

$$n = 0 \quad \text{or} \quad n = \frac{5}{2}$$

▶ The roots are $n = 0$ and $n = \frac{5}{2}$.

Write equation.

Factor left side.

Zero-Product Property

Solve for x .

Write equation.

Subtract $15n$ from each side.

Factor left side.

Zero-Product Property

Solve for n .

Solve the equation. Check your solutions.

8. $a^2 + 5a = 0$

9. $3s^2 - 9s = 0$

10. $4x^2 = 2x$

MONITORING PROGRESS ANSWERS

8. $a = 0, a = -5$

9. $s = 0, s = 3$

10. $x = 0, x = \frac{1}{2}$

11. 8 ft

Solving Real-Life Problems

EXAMPLE 5 Modeling with Mathematics

You can model the arch of a fireplace using the equation $y = -\frac{1}{9}(x + 18)(x - 18)$, where x and y are measured in inches. The x -axis represents the floor. Find the width of the arch at floor level.

