Chapter 7-5 Notes

- Factor $x^2 + bx + c$.
- Use factoring to solve real-life problems.

Factoring $x^2 + bx + c$ When c Is Positive

Algebra $x^2 + bx + c = (x + p)(x + q)$ when $p + q = b$ and $pq = c$.

When c is positive, p and q have the same sign as b.

Examples $x^2 + 6x + 5 = (x + 1)(x + 5)$

$x^2 - 6x + 5 = (x - 1)(x - 5)$

EXAMPLE 1 Factoring $x^2 + bx + c$ When b and c Are Positive

Factor $x^2 + 10x + 16$.

SOLUTION

Notice that $b = 10$ and $c = 16$.

- Because c is positive, the factors p and q must have the same sign so that pq is positive.
- Because b is also positive, p and q must each be positive so that $p + q$ is positive.

Find two positive integer factors of 16 whose sum is 10.

Factors of 16	Sum of factors
1, 16	17
2, 8	10
4, 4	8

The values of p and q are 2 and 8.

▶ So, $x^2 + 10x + 16 = (x + 2)(x + 8)$.

Factor the polynomial.

1. $x^2 + 7x + 6$
2. $x^2 + 9x + 8$

MONITORING PROGRESS ANSWERS

1. $(x + 1)(x + 6)$
2. $(x + 1)(x + 8)$

EXAMPLE 2 Factoring $x^2 + bx + c$ When b Is Negative and c Is Positive

Factor $x^2 - 8x + 12$.

SOLUTION

Notice that $b = -8$ and $c = 12$.

- Because c is positive, the factors p and q must have the same sign so that pq is positive.
- Because b is negative, p and q must each be negative so that $p + q$ is negative.

Find two negative integer factors of 12 whose sum is -8.

Factors of 12	$-1, -12$	$-2, -6$	$-3, -4$
Sum of factors	-13	-8	-7

The values of p and q are -2 and -6.

▶ So, $x^2 - 8x + 12 = (x - 2)(x - 6)$.

Core Concept

Factoring $x^2 + bx + c$ When c Is Negative

Algebra $x^2 + bx + c = (x + p)(x + q)$ when $p + q = b$ and $pq = c$.

When c is negative, p and q have different signs.

Example $x^2 - 4x - 5 = (x + 1)(x - 5)$

EXAMPLE 3 Factoring $x^2 + bx + c$ When c Is Negative

Factor $x^2 + 4x - 21$.

SOLUTION

Notice that $b = 4$ and $c = -21$. Because c is negative, the factors p and q must have different signs so that pq is negative.

Find two integer factors of -21 whose sum is 4.

Factors of −21	$-21, 1$	$-1, 21$	$-7, 3$	$-3, 7$
Sum of factors	-20	20	-4	4

The values of p and q are -3 and 7.

▶ So, $x^2 + 4x - 21 = (x - 3)(x + 7)$.

Factor the polynomial.

3. $w^2 - 4w + 3$

4. $n^2 - 12n + 35$

5. $x^2 - 14x + 24$

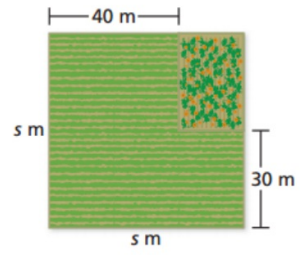
6. $x^2 + 2x - 15$

7. $y^2 + 13y - 30$

8. $v^2 - v - 42$

MONITORING PROGRESS ANSWERS

3. $(w - 1)(w - 3)$
4. $(n - 5)(n - 7)$
5. $(x - 2)(x - 12)$
6. $(x - 3)(x + 5)$
7. $(y - 2)(y + 15)$
8. $(v + 6)(v - 7)$



Solving Real-Life Problems

EXAMPLE 4 **Solving a Real-Life Problem**

A farmer plants a rectangular pumpkin patch in the northeast corner of a square plot of land. The area of the pumpkin patch is 600 square meters. What is the area of the square plot of land?

SOLUTION

SOLUTION

1. **Understand the Problem** You are given the area of the pumpkin patch, the difference of the side length of the square plot and the length of the pumpkin patch, and the difference of the side length of the square plot and the width of the pumpkin patch.

2. **Make a Plan** The length of the pumpkin patch is $(s - 30)$ meters and the width is $(s - 40)$ meters. Write and solve an equation to find the side length s. Then use the solution to find the area of the square plot of land.

3. **Solve the Problem** Use the equation for the area of a rectangle to write and solve an equation to find the side length s of the square plot of land.

$$600 = (s - 30)(s - 40) \qquad \text{Write an equation.}$$
$$600 = s^2 - 70s + 1200 \qquad \text{Multiply.}$$
$$0 = s^2 - 70s + 600 \qquad \text{Subtract 600 from each side.}$$
$$0 = (s - 10)(s - 60) \qquad \text{Factor the polynomial.}$$
$$s - 10 = 0 \quad \textit{or} \quad s - 60 = 0 \qquad \text{Zero-Product Property}$$
$$s = 10 \quad \textit{or} \quad s = 60 \qquad \text{Solve for } s.$$

▶ So, the area of the square plot of land is $60(60) = 3600$ square meters.