

## Skill check

Factor each polynomial.

1.  $2x^2 + 14x + 24$

## 7-7 Factoring Special Products

### What You Will Learn

- ▶ Factor the difference of two squares.
- ▶ Factor perfect square trinomials.
- ▶ Use factoring to solve real-life problems.

## Core Concept

### Difference of Two Squares Pattern

Algebra

$$a^2 - b^2 = (a + b)(a - b)$$

Example

$$x^2 - 9 = x^2 - 3^2 = (x + 3)(x - 3)$$

### EXAMPLE 1 Factoring the Difference of Two Squares

Factor (a)  $x^2 - 25$  and (b)  $4z^2 - 1$ .

### SOLUTION

a.  $x^2 - 25 = x^2 - 5^2$   
 $= (x + 5)(x - 5)$

▶ So,  $x^2 - 25 = (x + 5)(x - 5)$ .

b.  $4z^2 - 1 = (2z)^2 - 1^2$   
 $= (2z + 1)(2z - 1)$

▶ So,  $4z^2 - 1 = (2z + 1)(2z - 1)$ .

Write as  $a^2 - b^2$ .

Difference of two squares pattern

Write as  $a^2 - b^2$ .

Difference of two squares pattern

### EXAMPLE 2 Evaluating a Numerical Expression

Use a special product pattern to evaluate the expression  $54^2 - 48^2$ .

## SOLUTION

Notice that  $54^2 - 48^2$  is a difference of two squares. So, you can rewrite the expression in a form that it is easier to evaluate using the difference of two squares pattern.

$$54^2 - 48^2 = (54 + 48)(54 - 48)$$

Difference of two squares pattern

$$= 102(6)$$

Simplify.

$$= 612$$

Multiply.

► So,  $54^2 - 48^2 = 612$ .

**Factor the polynomial.**

1.  $x^2 - 36$

2.  $100 - m^2$

3.  $9n^2 - 16$

4.  $16h^2 - 49$

**Use a special product pattern to evaluate the expression.**

5.  $36^2 - 34^2$

6.  $47^2 - 44^2$

7.  $55^2 - 50^2$

8.  $28^2 - 24^2$

1.  $(x + 6)(x - 6)$
2.  $(10 + m)(10 - m)$
3.  $(3n + 4)(3n - 4)$
4.  $(4h + 7)(4h - 7)$
5. 140
6. 273
7. 525
8. 208

### Core Concept

#### Perfect Square Trinomial Pattern

##### Algebra

$$a^2 + 2ab + b^2 = (a + b)^2$$

$$a^2 - 2ab + b^2 = (a - b)^2$$

##### Example

$$\begin{aligned}x^2 + 6x + 9 &= x^2 + 2(x)(3) + 3^2 \\ &= (x + 3)^2\end{aligned}$$

$$\begin{aligned}x^2 - 6x + 9 &= x^2 - 2(x)(3) + 3^2 \\ &= (x - 3)^2\end{aligned}$$

**EXAMPLE 3** Factoring Perfect Square Trinomials

Factor each polynomial.

a.  $n^2 + 8n + 16$

b.  $4x^2 - 12x + 9$

### SOLUTION

a.  $n^2 + 8n + 16 = n^2 + 2(n)(4) + 4^2$   
 $= (n + 4)^2$

▶ So,  $n^2 + 8n + 16 = (n + 4)^2$ .

Write as  $a^2 + 2ab + b^2$ .

Perfect square trinomial pattern

b.  $4x^2 - 12x + 9 = (2x)^2 - 2(2x)(3) + 3^2$   
 $= (2x - 3)^2$

▶ So,  $4x^2 - 12x + 9 = (2x - 3)^2$ .

Write as  $a^2 - 2ab + b^2$ .

Perfect square trinomial pattern

### EXAMPLE 4

### Solving a Polynomial Equation

Solve  $x^2 + \frac{2}{3}x + \frac{1}{9} = 0$ .

**SOLUTION**

$$x^2 + \frac{2}{3}x + \frac{1}{9} = 0$$

$$9x^2 + 6x + 1 = 0$$

$$(3x)^2 + 2(3x)(1) + 1^2 = 0$$

$$(3x + 1)^2 = 0$$

$$3x + 1 = 0$$

$$x = -\frac{1}{3}$$

► The solution is  $x = -\frac{1}{3}$ .

Write equation.

Multiply each side by 9.

Write left side as  $a^2 + 2ab + b^2$ .

Perfect square trinomial pattern

Zero-Product Property

Solve for  $x$ .

**Factor the polynomial.**

9.  $m^2 - 2m + 1$

10.  $d^2 - 10d + 25$

11.  $9z^2 + 36z + 36$

**Solve the equation.**

12.  $a^2 + 6a + 9 = 0$

13.  $w^2 - \frac{7}{3}w + \frac{49}{36} = 0$

14.  $n^2 - 81 = 0$



9.  $(m - 1)^2$
10.  $(d - 5)^2$
11.  $9(z + 2)^2$
12.  $a = -3$
13.  $w = \frac{7}{6}$
14.  $n = -9, n = 9$


## Solving Real-Life Problems

### EXAMPLE 5 Modeling with Mathematics

A bird picks up a golf ball and drops it while flying. The function represents the height  $y$  (in feet) of the golf ball  $t$  seconds after it is dropped. The ball hits the top of a 32-foot-tall pine tree. After how many seconds does the ball hit the tree?



**1. Understand the Problem** You are given the height of the golf ball as a function of the amount of time after it is dropped and the height of the tree that the golf ball hits. You are asked to determine how many seconds it takes for the ball to hit the tree.


$$y = 81 - 16t^2$$

**2. Make a Plan** Use the function for the height of the golf ball. Substitute the height of the tree for  $y$  and solve for the time  $t$ .

**3. Solve the Problem** Substitute 32 for  $y$  and solve for  $t$ .

$$y = 81 - 16t^2$$

Write equation.

$$32 = 81 - 16t^2$$

Substitute 32 for  $y$ .

$$0 = 49 - 16t^2$$

Subtract 32 from each side.

$$0 = 7^2 - (4t)^2$$

Write as  $a^2 - b^2$ .

$$0 = (7 + 4t)(7 - 4t)$$

Difference of two squares pattern

$$7 + 4t = 0 \quad \text{or} \quad 7 - 4t = 0$$

Zero-Product Property

$$t = -\frac{7}{4} \quad \text{or} \quad t = \frac{7}{4}$$

Solve for  $t$ .

A negative time does not make sense in this situation.

► So, the golf ball hits the tree after  $\frac{7}{4}$ , or 1.75 seconds.