

## Skill check

A bird picks up a stone and drops it while flying. The function  $y = 49 - 16t^2$  represents the height  $y$  (in feet) of the stone  $t$  seconds after it is dropped. The stone hits the top of a 13-foot-tall building. After how many seconds does the stone hit the building? The stone hits the building after  $\frac{3}{2}$ , or 1.5, seconds.

## 7-8 Factoring Polynomials Completely

### What You Will Learn

- ▶ Factor polynomials by grouping.
- ▶ Factor polynomials completely.
- ▶ Use factoring to solve real-life problems.

## Core Concept

### Factoring by Grouping

To factor a polynomial with four terms, group the terms into pairs. Factor the GCF out of each pair of terms. Look for and factor out the common binomial factor.

This process is called **factoring by grouping**.

### EXAMPLE 1 Factoring by Grouping

Factor each polynomial by grouping.

a.  $x^3 + 3x^2 + 2x + 6$

b.  $x^2 + y + x + xy$

**SOLUTION**

a.  $x^3 + 3x^2 + 2x + 6 = (x^3 + 3x^2) + (2x + 6)$

Common binomial factor is  $x + 3$ .

$\rightarrow = x^2(x + 3) + 2(x + 3)$

$= (x + 3)(x^2 + 2)$

► So,  $x^3 + 3x^2 + 2x + 6 = (x + 3)(x^2 + 2)$ .

b.  $x^2 + y + x + xy = x^2 + x + xy + y$

$= (x^2 + x) + (xy + y)$

Common binomial factor is  $x + 1$ .

$\rightarrow = x(x + 1) + y(x + 1)$

$= (x + 1)(x + y)$

► So,  $x^2 + y + x + xy = (x + 1)(x + y)$ .

Group terms with common factors.

Factor out GCF of each pair of terms.

Factor out  $(x + 3)$ .

Rewrite polynomial.

Group terms with common factors.

Factor out GCF of each pair of terms.

Factor out  $(x + 1)$ .

**Factor the polynomial by grouping.**

1.  $a^3 + 3a^2 + a + 3$

2.  $y^2 + 2x + yx + 2y$

## MONITORING PROGRESS ANSWERS

1.  $(a^2 + 1)(a + 3)$
2.  $(y + 2)(y + x)$

### Guidelines for Factoring Polynomials Completely

To factor a polynomial completely, you should try each of these steps.

- |   |   |
|---|---|
| 1. Factor out the greatest common monomial factor.                                    | $3x^2 + 6x = 3x(x + 2)$                 |
| 2. Look for a difference of two squares or a perfect square trinomial.                | $x^2 + 4x + 4 = (x + 2)^2$              |
| 3. Factor a trinomial of the form $ax^2 + bx + c$ into a product of binomial factors. | $3x^2 - 5x - 2 = (3x + 1)(x - 2)$       |
| 4. Factor a polynomial with four terms by grouping.                                   | $x^3 + x - 4x^2 - 4 = (x^2 + 1)(x - 4)$ |

## EXAMPLE 2 Factoring Completely

Factor (a)  $3x^3 + 6x^2 - 18x$  and (b)  $7x^4 - 28x^2$ .

### SOLUTION

a.  $3x^3 + 6x^2 - 18x = 3x(x^2 + 2x - 6)$  Factor out  $3x$ .

$x^2 + 2x - 6$  is unfactorable, so the polynomial is factored completely.

▶ So,  $3x^3 + 6x^2 - 18x = 3x(x^2 + 2x - 6)$ .

b.  $7x^4 - 28x^2 = 7x^2(x^2 - 4)$  Factor out  $7x^2$ .

$= 7x^2(x^2 - 2^2)$  Write as  $a^2 - b^2$ .

$= 7x^2(x + 2)(x - 2)$  Difference of two squares pattern

▶ So,  $7x^4 - 28x^2 = 7x^2(x + 2)(x - 2)$ .

**EXAMPLE 3****Solving an Equation by Factoring Completely**

Solve  $2x^3 + 8x^2 = 10x$ .

**SOLUTION**

$$2x^3 + 8x^2 = 10x$$

$$2x^3 + 8x^2 - 10x = 0$$

$$2x(x^2 + 4x - 5) = 0$$

$$2x(x + 5)(x - 1) = 0$$

$$2x = 0 \quad \text{or} \quad x + 5 = 0 \quad \text{or} \quad x - 1 = 0$$

$$x = 0 \quad \text{or} \quad x = -5 \quad \text{or} \quad x = 1$$

► The roots are  $x = -5$ ,  $x = 0$ , and  $x = 1$ .

Original equation

Subtract  $10x$  from each side.

Factor out  $2x$ .

Factor  $x^2 + 4x - 5$ .

Zero-Product Property

Solve for  $x$ .

**Factor the polynomial completely.**

3.  $3x^3 - 12x$

4.  $2y^3 - 12y^2 + 18y$

5.  $m^3 - 2m^2 - 8m$

**Solve the equation.**

6.  $w^3 - 8w^2 + 16w = 0$

7.  $x^3 - 25x = 0$

8.  $c^3 - 7c^2 + 12c = 0$

**ANSWERS**

3.  $3x(x + 2)(x - 2)$

4.  $2y(y - 3)^2$

5.  $m(m + 2)(m - 4)$

6.  $w = 0, w = 4$

7.  $x = 0, x = -5, x = 5$

8.  $c = 0, c = 3, c = 4$

## Solving Real-Life Problems

### EXAMPLE 4 Modeling with Mathematics

A terrarium in the shape of a rectangular prism has a volume of 4608 cubic inches. Its length is more than 10 inches. The dimensions of the terrarium in terms of its width are shown. Find the length, width, and height of the terrarium.



$(36 - w)$  in.

$w$  in.

$(w + 4)$  in.

### SOLUTION

- 1. Understand the Problem** You are given the volume of a terrarium in the shape of a rectangular prism and a description of the length. The dimensions are written in terms of its width. You are asked to find the length, width, and height of the terrarium.
- 2. Make a Plan** Use the formula for the volume of a rectangular prism to write and solve an equation for the width of the terrarium. Then substitute that value in the expressions for the length and height of the terrarium.
- 3. Solve the Problem**

$$\text{Volume} = \text{length} \cdot \text{width} \cdot \text{height}$$

$$4608 = (36 - w)(w)(w + 4)$$

$$4608 = 32w^2 + 144w - w^3$$

$$0 = 32w^2 + 144w - w^3 - 4608$$

$$0 = (-w^3 + 32w^2) + (144w - 4608)$$

$$0 = -w^2(w - 32) + 144(w - 32)$$

$$0 = (w - 32)(-w^2 + 144)$$

$$0 = -1(w - 32)(w^2 - 144)$$

$$0 = -1(w - 32)(w - 12)(w + 12)$$

$$w - 32 = 0 \quad \text{or} \quad w - 12 = 0 \quad \text{or} \quad w + 12 = 0$$

$$w = 32 \quad \text{or} \quad w = 12 \quad \text{or} \quad w = -12$$

Volume of a rectangular prism

Write equation.

Multiply.

Subtract 4608 from each side.

Group terms with common factors.

Factor out GCF of each pair of terms.

Factor out  $(w - 32)$ .

Factor  $-1$  from  $-w^2 + 144$

Difference of two squares pattern

Zero-Product Property

Solve for  $w$ .



9. A box in the shape of a rectangular prism has a volume of 72 cubic feet. The box has a length of  $x$  feet, a width of  $(x - 1)$  feet, and a height of  $(x + 9)$  feet. Find the dimensions of the box.

9. length: 3 ft, width: 2 ft, height: 12 ft

