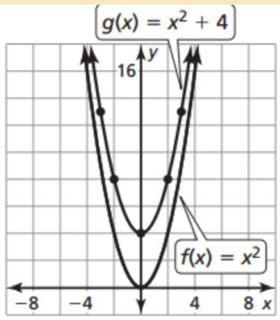
Skill check

Graph the function. Compare the graph to the graph of $f(x) = x^2$.

1. $g(x) = x^2 + 4$



The graph of g is a vertical translation 4 units up of the graph of f.

8.3 Graphing $f(x) = ax^2 + bx + c$

EXPLORATION 1 **Comparing x-Intercepts with the Vertex**

Work with a partner.

a. Sketch the graphs of $y = 2x^2 - 8x$ and $y = 2x^2 - 8x + 6$.

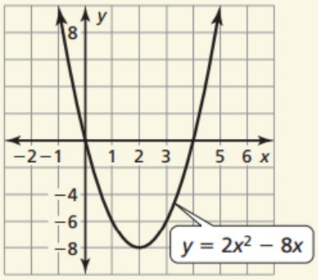
b. What do you notice about the x-coordinate of the vertex of each graph?

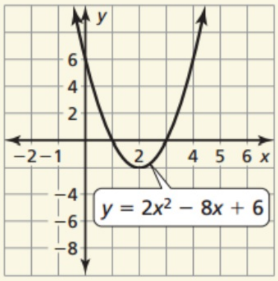
c. Use the graph of $y = 2x^2 - 8x$ to find its x-intercepts. Verify your answer by solving $0 = 2x^2 - 8x$.

d. Compare the value of the x-coordinate of the vertex with the values of the x-intercepts.

ANSWERS

1. a.





b. They are the same.

c. $x = 0, x = 4$

d. *Sample answer:* The value of the x-coordinate of the vertex is the average of the values of the x-intercepts.

What You Will Learn

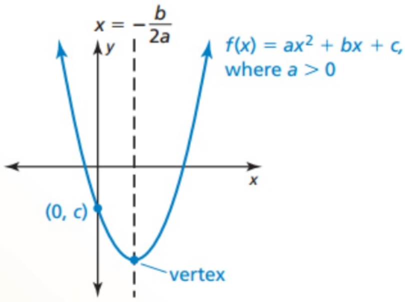
- Graph quadratic functions of the form $f(x) = ax^2 + bx + c$.
- Find maximum and minimum values of quadratic functions.

Graphing $f(x) = ax^2 + bx + c$

Core Concept

Graphing $f(x) = ax^2 + bx + c$

- The graph opens up when $a > 0$, and the graph opens down when $a < 0$.
- The y-intercept is c.
- The x-coordinate of the vertex is $-\frac{b}{2a}$.
- The axis of symmetry is $x = -\frac{b}{2a}$.



EXAMPLE 1 **Finding the Axis of Symmetry and the Vertex**

Find (a) the axis of symmetry and (b) the vertex of the graph of $f(x) = 2x^2 + 8x - 1$.

SOLUTION

a. Find the axis of symmetry when $a = 2$ and $b = 8$.

$x = -\frac{b}{2a}$	Write the equation for the axis of symmetry.
$x = -\frac{8}{2(2)}$	Substitute 2 for *a* and 8 for *b*.
$x = -2$	Simplify.

▶ The axis of symmetry is $x = -2$.

b. The axis of symmetry is $x = -2$, so the *x*-coordinate of the vertex is -2. Use the function to find the *y*-coordinate of the vertex.

$f(x) = 2x^2 + 8x - 1$	Write the function.
$f(-2) = 2(-2)^2 + 8(-2) - 1$	Substitute -2 for *x*.
$= -9$	Simplify.

▶ The vertex is $(-2, -9)$.

Find (a) the axis of symmetry and (b) the vertex of the graph of the function.

1. $f(x) = 3x^2 - 2x$ **2.** $g(x) = x^2 + 6x + 5$ **3.** $h(x) = -\frac{1}{2}x^2 + 7x - 4$

1. a. $x = \frac{1}{3}$

   b. $\left(\frac{1}{3}, -\frac{1}{3}\right)$

2. a. $x = -3$

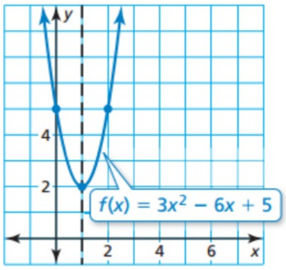
   b. $(-3, -4)$

3. a. $x = 7$

   b. $\left(7, \frac{41}{2}\right)$

EXAMPLE 2 **Graphing $f(x) = ax^2 + bx + c$**

Graph $f(x) = 3x^2 - 6x + 5$. Describe the domain and range.



SOLUTION

Step 1 Find and graph the axis of symmetry.

$$x = -\frac{b}{2a} = -\frac{(-6)}{2(3)} = 1 \qquad \text{Substitute and simplify.}$$

Step 2 Find and plot the vertex.

The axis of symmetry is $x = 1$, so the x-coordinate of the vertex is 1. Use the function to find the y-coordinate of the vertex.

$f(x) = 3x^2 - 6x + 5$ — Write the function.

$f(1) = 3(1)^2 - 6(1) + 5$ — Substitute 1 for x.

$= 2$ — Simplify.

So, the vertex is (1, 2).

Step 3 Use the y-intercept to find two more points on the graph.

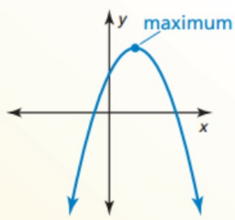
Because $c = 5$, the y-intercept is 5. So, (0, 5) lies on the graph. Because the axis of symmetry is $x = 1$, the point (2, 5) also lies on the graph.

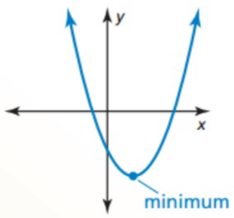
Step 4 Draw a smooth curve through the points.

Maximum and Minimum Values

The y-coordinate of the vertex of the graph of $f(x) = ax^2 + bx + c$ is the **maximum value** of the function when $a < 0$ or the **minimum value** of the function when $a > 0$.

$f(x) = ax^2 + bx + c, a < 0$ $\qquad$ $f(x) = ax^2 + bx + c, a > 0$





EXAMPLE 3 Finding a Maximum or Minimum Value

Tell whether the function $f(x) = -4x^2 - 24x - 19$ has a minimum value or a maximum value. Then find the value.

SOLUTION

For $f(x) = -4x^2 - 24x - 19$, $a = -4$ and $-4 < 0$. So, the parabola opens down and the function has a maximum value. To find the maximum value, find the y-coordinate of the vertex.

First, find the x-coordinate of the vertex. Use $a = -4$ and $b = -24$.

$$x = -\frac{b}{2a} = -\frac{(-24)}{2(-4)} = -3$$ Substitute and simplify.

Then evaluate the function when $x = -3$ to find the y-coordinate of the vertex.

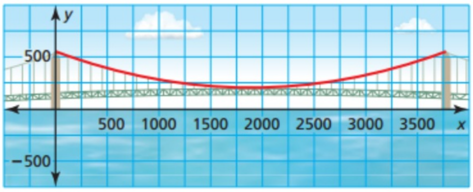
$$f(-3) = -4(-3)^2 - 24(-3) - 19$$ Substitute −3 for *x*.

$$= 17$$ Simplify.

▶ The maximum value is 17.

EXAMPLE 4 **Finding a Minimum Value**

The suspension cables between the two towers of the Mackinac Bridge in Michigan form a parabola that can be modeled by $y = 0.000098x^2 - 0.37x + 552$, where x and y are measured in feet. What is the height of the cable above the water at its lowest point?



SOLUTION

The lowest point of the cable is at the vertex of the parabola. Find the x-coordinate of the vertex. Use $a = 0.000098$ and $b = -0.37$.

$$x = -\frac{b}{2a} = -\frac{(-0.37)}{2(0.000098)} \approx 1888$$ Substitute and use a calculator.

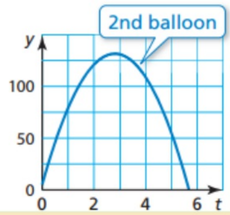
Substitute 1888 for x in the equation to find the y-coordinate of the vertex.

$$y = 0.000098(1888)^2 - 0.37(1888) + 552 \approx 203$$

▶ The cable is about 203 feet above the water at its lowest point.

EXAMPLE 5 Modeling with Mathematics

A group of friends is launching water balloons. The function $f(t) = -16t^2 + 80t + 5$ represents the height (in feet) of the first water balloon t seconds after it is launched. The height of the second water balloon t seconds after it is launched is shown in the graph. Which water balloon went higher?

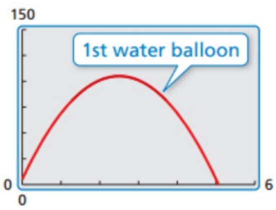


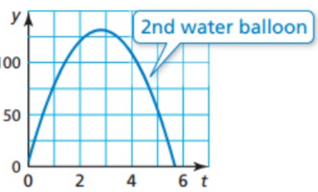
SOLUTION

1. **Understand the Problem** You are given a function that represents the height of the first water balloon. The height of the second water balloon is represented graphically. You need to find and compare the maximum heights of the water balloons.

2. **Make a Plan** To compare the maximum heights, represent both functions graphically. Use a graphing calculator to graph $f(t) = -16t^2 + 80t + 5$ in an appropriate viewing window. Then visually compare the heights of the water balloons.

3. **Solve the Problem** Enter the function $f(t) = -16t^2 + 80t + 5$ into your calculator and graph it. Compare the graphs to determine which function has a greater maximum value.





You can see that the second water balloon reaches a height of about 125 feet, while the first water balloon reaches a height of only about 100 feet.

▶ So, the second water balloon went higher.

4. **Look Back** Use the *maximum* feature to determine that the maximum value of $f(t) = -16t^2 + 80t + 5$ is 105. Use a straightedge to represent a height of 105 feet on the graph that represents the second water balloon to clearly see that the second water balloon went higher.