8.5 Lesson

Core Vocabulary

intercept form, *p. 450*

What You Will Learn

- Graph quadratic functions of the form $f(x) = a(x - p)(x - q)$.
- Use intercept form to find zeros of functions.
- Use characteristics to graph and write quadratic functions.
- Use characteristics to graph and write cubic functions.

Graphing $f(x) = a(x - p)(x - q)$

You have already graphed quadratic functions written in several different forms, such as $f(x) = ax^2 + bx + c$ (standard form) and $g(x) = a(x - h)^2 + k$ (vertex form). Quadratic functions can also be written in **intercept form**, $f(x) = a(x - p)(x - q)$, where $a \neq 0$. In this form, the polynomial that defines a function is in factored form and the x-intercepts of the graph can be easily determined.

3 different forms:

Standard:

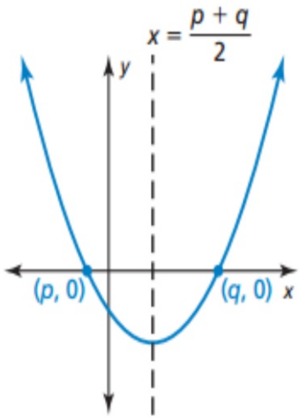
Vertex:

Intercept:

Core Concept

Graphing $f(x) = a(x - p)(x - q)$

- The x-intercepts are p and q.
- The axis of symmetry is halfway between $(p, 0)$ and $(q, 0)$. So, the axis of symmetry is $x = \frac{p + q}{2}$.
- The graph opens up when $a > 0$, and the graph opens down when $a < 0$.



EXAMPLE 1 **Graphing $f(x) = a(x - p)(x - q)$**

Graph $f(x) = -(x + 1)(x - 5)$. Describe the domain and range.

SOLUTION

Step 1 Identify the x-intercepts. Because the x-intercepts are $p = -1$ and $q = 5$, plot $(-1, 0)$ and $(5, 0)$.

Step 2 Find and graph the axis of symmetry.

$$x = \frac{p + q}{2} = \frac{-1 + 5}{2} = 2$$

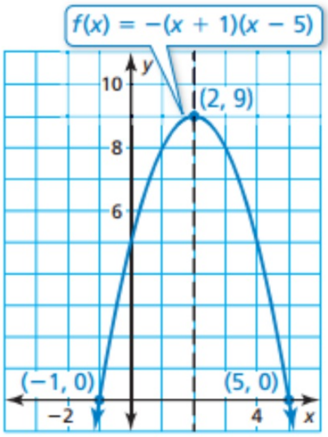
Step 3 Find and plot the vertex.

The x-coordinate of the vertex is 2. To find the y-coordinate of the vertex, substitute 2 for x and simplify.

$$f(2) = -(2 + 1)(2 - 5) = 9$$

So, the vertex is $(2, 9)$.

Step 4 Draw a parabola through the vertex and the points where the x-intercepts occur.



▶ The domain is all real numbers. The range is $y \leq 9$.

EXAMPLE 2 Graphing a Quadratic Function

Graph $f(x) = 2x^2 - 8$. Describe the domain and range.

SOLUTION

Step 1 Rewrite the quadratic function in intercept form.

$f(x) = 2x^2 - 8$ — Write the function.

$= 2(x^2 - 4)$ — Factor out common factor.

$= 2(x + 2)(x - 2)$ — Difference of two squares pattern

Step 2 Identify the x-intercepts. Because the x-intercepts are $p = -2$ and $q = 2$, plot $(-2, 0)$ and $(2, 0)$.

Step 3 Find and graph the axis of symmetry.

$$x = \frac{p + q}{2} = \frac{-2 + 2}{2} = 0$$

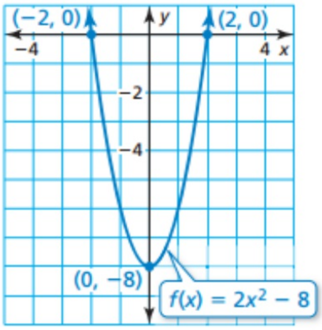
Step 4 Find and plot the vertex.

The x-coordinate of the vertex is 0.
The y-coordinate of the vertex is

$f(0) = 2(0)^2 - 8 = -8.$

So, the vertex is $(0, -8)$.

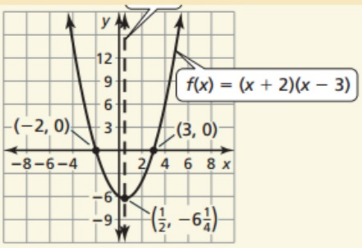
Step 5 Draw a parabola through the vertex and the points where the x-intercepts occur.



▶ The domain is all real numbers. The range is $y \geq -8$.

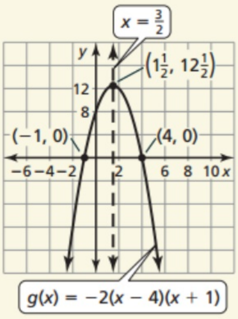
Graph the quadratic function. Label the vertex, axis of symmetry, and x-intercepts. Describe the domain and range of the function.

1. $f(x) = (x + 2)(x - 3)$ **2.** $g(x) = -2(x - 4)(x + 1)$ **3.** $h(x) = 4x^2 - 36$



domain: all real numbers,
range: $y \geq -6\frac{1}{4}$

2.



domain: all real numbers,
range: $y \leq 12\frac{1}{2}$

Factors and Zeros

For any factor $x - n$ of a polynomial, n is a zero of the function defined by the polynomial.

Find the zeros of each function.

a. $f(x) = -2x^2 - 10x - 12$

b. $h(x) = (x - 1)(x^2 - 16)$

a. $f(x) = -2x^2 - 10x - 12$ Write the function.

$= -2(x^2 + 5x + 6)$ Factor out common factor.

$= -2(x + 3)(x + 2)$ Factor the trinomial.

▶ So, the zeros of the function are -3 and -2.

b. $h(x) = (x - 1)(x^2 - 16)$ Write the function.

$= (x - 1)(x + 4)(x - 4)$ Difference of two squares pattern

▶ So, the zeros of the function are -4, 1, and 4.

Find the zero(s) of the function.

4. $f(x) = (x - 6)(x - 1)$ **5.** $g(x) = 3x^2 - 12x + 12$ **6.** $h(x) = x(x^2 - 1)$

MONITORING PROGRESS ANSWERS

4. 1, 6

5. 2

6. −1, 0, 1

Using Characteristics to Graph and Write Quadratic Functions

EXAMPLE 5 **Graphing a Quadratic Function Using Zeros**

Use zeros to graph $h(x) = x^2 - 2x - 3$.

SOLUTION

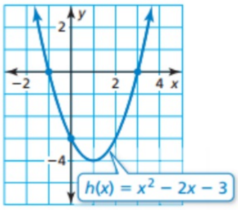
The function is in standard form. You know that the parabola opens up ($a > 0$) and the y-intercept is -3. So, begin by plotting $(0, -3)$.

Notice that the polynomial that defines the function is factorable. So, write the function in intercept form and identify the zeros.

$h(x) = x^2 - 2x - 3$ Write the function.

$= (x + 1)(x - 3)$ Factor the trinomial.

The zeros of the function are -1 and 3. So, plot $(-1, 0)$ and $(3, 0)$. Draw a parabola through the points.



EXAMPLE 6 **Writing Quadratic Functions**

Write a quadratic function in standard form whose graph satisfies the given condition(s).

a. vertex: $(-3, 4)$

b. passes through $(-9, 0)$, $(-2, 0)$, and $(-4, 20)$

a. Because you know the vertex, use vertex form to write a function.

$f(x) = a(x - h)^2 + k$	Vertex form
$= 1(x + 3)^2 + 4$	Substitute for a, h, and k.
$= x^2 + 6x + 9 + 4$	Find the product $(x + 3)^2$.
$= x^2 + 6x + 13$	Combine like terms.

b. The given points indicate that the x-intercepts are -9 and -2. So, use intercept form to write a function.

$f(x) = a(x - p)(x - q)$	Intercept form
$= a(x + 9)(x + 2)$	Substitute for p and q.

Use the other given point, $(-4, 20)$, to find the value of a.

$20 = a(-4 + 9)(-4 + 2)$	Substitute -4 for x and 20 for $f(x)$.
$20 = a(5)(-2)$	Simplify.
$-2 = a$	Solve for a.

Use the value of a to write the function.

$f(x) = -2(x + 9)(x + 2)$	Substitute -2 for a.
$= -2x^2 - 22x - 36$	Simplify.

SOLUTION

From the graph, you can see that the x-intercepts are 0, 2, and 5. Use intercept form to write a function.

$f(x) = a(x - p)(x - q)(x - r)$ — Intercept form

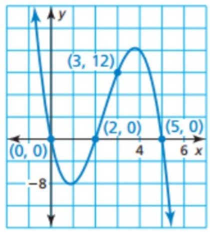
$= a(x - 0)(x - 2)(x - 5)$ — Substitute for p, q, and r.

$= a(x)(x - 2)(x - 5)$ — Simplify.

Use the other given point, (3, 12), to find the value of a.

$12 = a(3)(3 - 2)(3 - 5)$ — Substitute 3 for x and 12 for $f(x)$.

$-2 = a$ — Solve for a.



Use the value of a to write the function.

$f(x) = -2(x)(x - 2)(x - 5)$ — Substitute −2 for a.

$= -2x^3 + 14x^2 - 20x$ — Simplify.

▶ The function represented by the graph is $f(x) = -2x^3 + 14x^2 - 20x$.