

9.1 Properties of Radicals

- ▶ Use properties of radicals to simplify expressions.
- ▶ Simplify expressions by rationalizing the denominator.
- ▶ Perform operations with radicals.

Product Property of Square Roots

Words The square root of a product equals the product of the square roots of the factors.

Numbers $\sqrt{9 \cdot 5} = \sqrt{9} \cdot \sqrt{5} = 3\sqrt{5}$

Algebra $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$, where $a, b \geq 0$

EXAMPLE 1 Using the Product Property of Square Roots

a. $\sqrt{108} = \sqrt{36 \cdot 3}$

b. $\sqrt{9x^3} = \sqrt{9 \cdot x^2 \cdot x}$

a. $\sqrt{108} = \sqrt{36 \cdot 3}$

$$= \sqrt{36} \cdot \sqrt{3}$$

$$= 6\sqrt{3}$$

b. $\sqrt{9x^3} = \sqrt{9 \cdot x^2 \cdot x}$

$$= \sqrt{9} \cdot \sqrt{x^2} \cdot \sqrt{x}$$

$$= 3x\sqrt{x}$$

Factor using the greatest perfect square factor.

Product Property of Square Roots

Simplify.

Factor using the greatest perfect square factor.

Product Property of Square Roots

Simplify.

Simplify the expression.

1. $\sqrt{24}$

2. $-\sqrt{80}$

3. $\sqrt{49x^3}$

4. $\sqrt{75n^5}$

MONITORING PROGRESS ANSWERS

1. $2\sqrt{6}$
2. $-4\sqrt{5}$
3. $7x\sqrt{x}$
4. $5n^2\sqrt{3n}$

Quotient Property of Square Roots

Words The square root of a quotient equals the quotient of the square roots of the numerator and denominator.

Numbers $\sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{\sqrt{4}} = \frac{\sqrt{3}}{2}$ **Algebra** $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$, where $a \geq 0$ and $b > 0$

EXAMPLE 2 Using the Quotient Property of Square Roots

a. $\sqrt{\frac{15}{64}} = \frac{\sqrt{15}}{\sqrt{64}}$

Quotient Property of Square Roots

b.
$$\sqrt{\frac{81}{x^2}} = \frac{\sqrt{81}}{\sqrt{x^2}}$$
$$= \frac{9}{x}$$

Performing Operations with Radicals

Radicals with the same index and radicand are called **like radicals**. You can add and subtract like radicals the same way you combine like terms by using the Distributive Property.

EXAMPLE 8 Adding and Subtracting Radicals

- a. $5\sqrt{7} + \sqrt{11} - 8\sqrt{7} = 5\sqrt{7} - 8\sqrt{7} + \sqrt{11}$ Commutative Property of Addition
 $= (5 - 8)\sqrt{7} + \sqrt{11}$ Distributive Property
 $= -3\sqrt{7} + \sqrt{11}$ Subtract.
- b. $10\sqrt{5} + \sqrt{20} = 10\sqrt{5} + \sqrt{4 \cdot 5}$ Factor using the greatest perfect square factor.
 $= 10\sqrt{5} + \sqrt{4} \cdot \sqrt{5}$ Product Property of Square Roots
 $= 10\sqrt{5} + 2\sqrt{5}$ Simplify.
 $= (10 + 2)\sqrt{5}$ Distributive Property
 $= 12\sqrt{5}$ Add.
- c. $6\sqrt[3]{x} + 2\sqrt[3]{x} = (6 + 2)\sqrt[3]{x}$ Distributive Property
 $= 8\sqrt[3]{x}$ Add.

EXAMPLE 9 Multiplying RadicalsSimplify $\sqrt{5}(\sqrt{3} - \sqrt{75})$.**SOLUTION**

Method 1	$\sqrt{5}(\sqrt{3} - \sqrt{75}) = \sqrt{5} \cdot \sqrt{3} - \sqrt{5} \cdot \sqrt{75}$	Distributive Property
	$= \sqrt{15} - \sqrt{375}$	Product Property of Square Roots
	$= \sqrt{15} - 5\sqrt{15}$	Simplify.
	$= (1 - 5)\sqrt{15}$	Distributive Property
	$= -4\sqrt{15}$	Subtract.
Method 2	$\sqrt{5}(\sqrt{3} - \sqrt{75}) = \sqrt{5}(\sqrt{3} - 5\sqrt{3})$	Simplify $\sqrt{75}$.
	$= \sqrt{5}[(1 - 5)\sqrt{3}]$	Distributive Property
	$= \sqrt{5}(-4\sqrt{3})$	Subtract.
	$= -4\sqrt{15}$	Product Property of Square Roots

Simplify the expression.

23. $3\sqrt{2} - \sqrt{6} + 10\sqrt{2}$

25. $4\sqrt[3]{5x} - 11\sqrt[3]{5x}$

27. $(2\sqrt{5} - 4)^2$

24. $4\sqrt{7} - 6\sqrt{63}$

26. $\sqrt{3}(8\sqrt{2} + 7\sqrt{32})$

28. $\sqrt[3]{-4}(\sqrt[3]{2} - \sqrt[3]{16})$

23. $13\sqrt{2} - \sqrt{6}$

24. $-14\sqrt{7}$

25. $-7\sqrt[3]{5x}$

26. $36\sqrt{6}$

27. $36 - 16\sqrt{5}$

28. 2