9.3 Solving Quadratic Equations Using Square Roots

- Solve quadratic equations using square roots.
- Approximate the solutions of quadratic equations.

Solutions of $x^2 = d$

- When $d > 0$, $x^2 = d$ has two real solutions, $x = \pm\sqrt{d}$.
- When $d = 0$, $x^2 = d$ has one real solution, $x = 0$.
- When $d < 0$, $x^2 = d$ has no real solutions.

EXAMPLE 1 **Solving Quadratic Equations Using Square Roots**

a. Solve $3x^2 - 27 = 0$ using square roots.

$3x^2 - 27 = 0$	Write the equation.
$3x^2 = 27$	Add 27 to each side.
$x^2 = 9$	Divide each side by 3.
$x = \pm\sqrt{9}$	Take the square root of each side.
$x = \pm 3$	Simplify.

The solutions are $x = 3$ and $x = -3$.

You can also solve $3x^2 - 27 = 0$ by factoring.

$$3(x^2 - 9) = 0$$
$$3(x - 3)(x + 3) = 0$$
$$x = 3 \text{ or } x = -3$$

b. Solve $x^2 - 10 = -10$ using square roots.

$x^2 - 10 = -10$	Write the equation.
$x^2 = 0$	Add 10 to each side.
$x = 0$	Take the square root of each side.

▶ The only solution is $x = 0$.

c. Solve $-5x^2 + 11 = 16$ using square roots.

$-5x^2 + 11 = 16$	Write the equation.
$-5x^2 = 5$	Subtract 11 from each side.
$x^2 = -1$	Divide each side by -5.

▶ The square of a real number cannot be negative. So, the equation has no real solutions.

EXAMPLE 2 Solving a Quadratic Equation Using Square Roots

Solve $(x - 1)^2 = 25$ using square roots.

Each side of the equation $(x - 1)^2 = 25$ is a square. So, you can still solve by taking the square root of each side.

Solve $(x - 1)^2 = 25$ using square roots.

SOLUTION

$(x - 1)^2 = 25$ Write the equation.

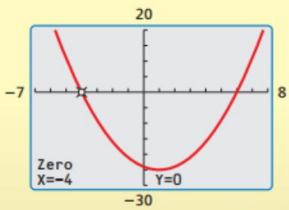
$x - 1 = \pm 5$ Take the square root of each side.

$x = 1 \pm 5$ Add 1 to each side.

▶ So, the solutions are $x = 1 + 5 = 6$ and $x = 1 - 5 = -4$.

Check

Use a graphing calculator to check your answer. Rewrite the equation as $(x - 1)^2 - 25 = 0$. Graph the related function $f(x) = (x - 1)^2 - 25$ and find the zeros of the function. The zeros are -4 and 6.



Solve the equation using square roots.

1. $-3x^2 = -75$
2. $x^2 + 12 = 10$
3. $4x^2 - 15 = -15$
4. $(x + 7)^2 = 0$
5. $4(x - 3)^2 = 9$
6. $(2x + 1)^2 = 36$

ANSWERS

1. $x = 5, x = -5$
2. no real solutions
3. $x = 0$
4. $x = -7$
5. $x = \frac{9}{2}, x = \frac{3}{2}$
6. $x = \frac{5}{2}, x = -\frac{7}{2}$

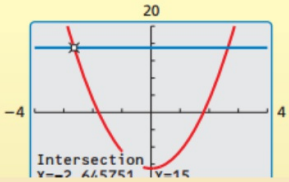
Approximating Solutions of Quadratic Equations

EXAMPLE 3 Approximating Solutions of a Quadratic Equation

Solve $4x^2 - 13 = 15$ using square roots. Round the solutions to the nearest hundredth.

Check

Graph each side of the equation and find the points of intersection. The x-values of the points of intersection are about -2.65 and 2.65.



Solve $4x^2 - 13 = 15$ using square roots. Round the solutions to the

SOLUTION

$4x^2 - 13 = 15$	Write the equation.
$4x^2 = 28$	Add 13 to each side.
$x^2 = 7$	Divide each side by 4.
$x = \pm\sqrt{7}$	Take the square root of each side.
$x \approx \pm 2.65$	Use a calculator.

▶ The solutions are $x \approx -2.65$ and $x \approx 2.65$.

Solve the equation using square roots. Round your solutions to the nearest hundredth.

7. $x^2 + 8 = 19$ **8.** $5x^2 - 2 = 0$ **9.** $3x^2 - 30 = 4$

7. $x \approx 3.32, x \approx -3.32$

8. $x \approx 0.63, x \approx -0.63$

9. $x \approx 3.37, x \approx -3.37$

EXAMPLE 4 Solving a Real-Life Problem

A touch tank has a height of 3 feet. Its length is three times its width. The volume of the tank is 270 cubic feet. Find the length and width of the tank.

SOLUTION

The length ℓ is three times the width w, so $\ell = 3w$. Write an equation using the formula for the volume of a rectangular prism.

$V = \ell wh$	Write the formula.
$270 = 3w(w)(3)$	Substitute 270 for V, $3w$ for ℓ, and 3 for h.
$270 = 9w^2$	Multiply.
$30 = w^2$	Divide each side by 9.
$\pm\sqrt{30} = w$	Take the square root of each side.

The solutions are $\sqrt{30}$ and $-\sqrt{30}$. Use the positive solution.

So, the width is $\sqrt{30} \approx 5.5$ feet and the length is $3\sqrt{30} \approx 16.4$ feet.

INTERPRETING MATHEMATICAL RESULTS

Use the positive square root because negative solutions do not make sense in this context. Length and width cannot be negative.