What You Will Learn

- Solve quadratic equations using the Quadratic Formula.
- Interpret the discriminant.
- Choose efficient methods for solving quadratic equations.

Using the Quadratic Formula

By completing the square for the quadratic equation $ax^2 + bx + c = 0$, you can develop a formula that gives the solutions of any quadratic equation in standard form. This formula is called the **Quadratic Formula**.

Song

EXAMPLE 1 Using the Quadratic Formula

Solve $2x^2 - 5x + 3 = 0$ using the Quadratic Formula.

SOLUTION

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ Quadratic Formula

$$= \frac{-(-5) \pm \sqrt{(-5)^2 - 4(2)(3)}}{2(2)}$$ Substitute 2 for a, -5 for b, and 3 for c.

$$= \frac{5 \pm \sqrt{1}}{4}$$ Simplify.

$$= \frac{5 \pm 1}{4}$$ Evaluate the square root.

So, the solutions are $x = \frac{5 + 1}{4} = \frac{3}{2}$ and $x = \frac{5 - 1}{4} = 1$.

Solve the equation using the Quadratic Formula. Round your solutions to the nearest tenth, if necessary.

1. $x^2 - 6x + 5 = 0$

2. $\frac{1}{2}x^2 + x - 10 = 0$

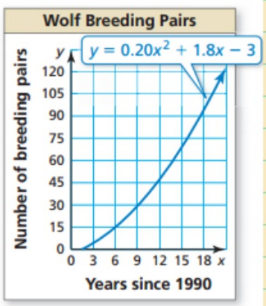
3. $-3x^2 + 2x + 7 = 0$

4. $4x^2 - 4x = -1$

1. $x = 5, x = 1$
2. $x \approx 3.6, x \approx -5.6$
3. $x \approx -1.2, x \approx 1.9$
4. $x = \frac{1}{2}$

EXAMPLE 2 Modeling With Mathematics

The number y of Northern Rocky Mountain wolf breeding pairs x years since 1990 can be modeled by the function $y = 0.20x^2 + 1.8x - 3$. When were there about 35 breeding pairs?



SOLUTION

1. **Understand the Problem** You are given a quadratic function that represents the number of wolf breeding pairs for years after 1990. You need to use the model to determine when there were 35 wolf breeding pairs.

2. **Make a Plan** To determine when there were 35 wolf breeding pairs, find the x-values for which $y = 35$. So, solve the equation $35 = 0.20x^2 + 1.8x - 3$.

3. **Solve the Problem**

$$35 = 0.20x^2 + 1.8x - 3 \quad \text{Write the equation.}$$

$$0 = 0.20x^2 + 1.8x - 38 \quad \text{Write in standard form.}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \text{Quadratic Formula}$$

$$= \frac{-1.8 \pm \sqrt{1.8^2 - 4(0.2)(-38)}}{2(0.2)} \quad \text{Substitute 0.2 for } a\text{, 1.8 for } b\text{, and } -38 \text{ for } c.$$

$$= \frac{-1.8 \pm \sqrt{33.64}}{0.4} \quad \text{Simplify.}$$

$$= \frac{-1.8 \pm 5.8}{0.4} \quad \text{Simplify.}$$

The solutions are $x = \frac{-1.8 + 5.8}{0.4} = 10$ and $x = \frac{-1.8 - 5.8}{0.4} = -19$.

▶ Because x represents the number of years since 1990, x is greater than or equal to zero. So, there were about 35 breeding pairs 10 years after 1990, in 2000.

Interpreting the Discriminant

The expression $b^2 - 4ac$ in the Quadratic Formula is called the **discriminant**.

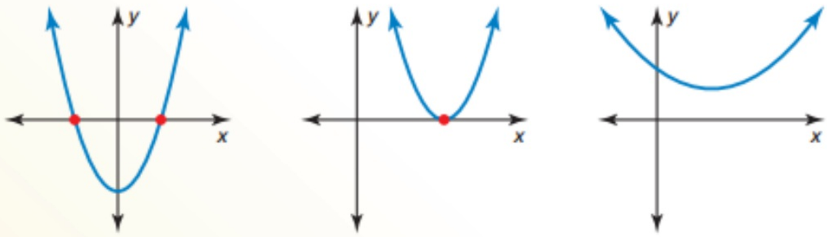
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

← discriminant

Because the discriminant is under the radical symbol, you can use the value of the discriminant to determine the number of real solutions of a quadratic equation and the number of x-intercepts of the graph of the related function.

Interpreting the Discriminant

$b^2 - 4ac > 0$	$b^2 - 4ac = 0$	$b^2 - 4ac < 0$



• two real solutions • two x-intercepts	• one real solution • one x-intercept	• no real solutions • no x-intercepts

EXAMPLE 3 **Determining the Number of Real Solutions**

a. Determine the number of real solutions of $x^2 + 8x - 3 = 0$.

$$b^2 - 4ac = 8^2 - 4(1)(-3)$$ Substitute 1 for *a*, 8 for *b*, and −3 for *c*.

$$= 64 + 12$$ Simplify.

$$= 76$$ Add.

▶ The discriminant is greater than 0. So, the equation has two real solutions.

b. Determine the number of real solutions of $9x^2 + 1 = 6x$.

Write the equation in standard form: $9x^2 - 6x + 1 = 0$.

Write the equation in standard form: $9x^2 - 6x + 1 = 0$.

$$b^2 - 4ac = (-6)^2 - 4(9)(1)$$ Substitute 9 for *a*, −6 for *b*, and 1 for *c*.

$$= 36 - 36$$ Simplify.

$$= 0$$ Subtract.

▶ The discriminant is 0. So, the equation has one real solution.

7. $-x^2 + 4x - 4 = 0$

8. $6x^2 + 2x = -1$

9. $\frac{1}{2}x^2 = 7x - 1$

EXAMPLE 4 Finding the Number of x-Intercepts of a Parabola

Find the number of x-intercepts of the graph of $y = 2x^2 + 3x + 9$.

Determine the number of real solutions of $0 = 2x^2 + 3x + 9$.

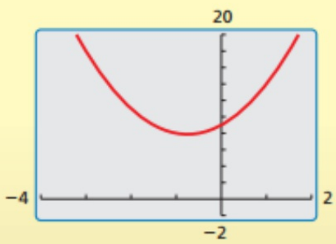
$$\begin{aligned} b^2 - 4ac &= 3^2 - 4(2)(9) && \text{Substitute 2 for } a\text{, 3 for } b\text{, and 9 for } c. \\ &= 9 - 72 && \text{Simplify.} \\ &= -63 && \text{Subtract.} \end{aligned}$$

Because the discriminant is less than 0, the equation has no real solutions.

▶ So, the graph of $y = 2x^2 + 3x + 9$ has no x-intercepts.

Check

Use a graphing calculator to check your answer. Notice that the graph of $y = 2x^2 + 3x + 9$ has no x-intercepts.



Find the number of x-intercepts of the graph of the function.

10. $y = -x^2 + x - 6$ **11.** $y = x^2 - x$ **12.** $f(x) = x^2 + 12x + 36$

10. no x-intercepts

11. two x-intercepts

12. one x-intercept

Methods for Solving Quadratic Equations

Method	Advantages	Disadvantages
Factoring *(Lessons 7.5–7.8)*	• Straightforward when the equation can be factored easily	• Some equations are not factorable.
Graphing *(Lesson 9.2)*	• Can easily see the number of solutions • Use when approximate solutions are sufficient. • Can use a graphing calculator	• May not give exact solutions
Using Square Roots *(Lesson 9.3)*	• Use to solve equations of the form $x^2 = d$.	• Can only be used for certain equations
Completing the Square *(Lesson 9.4)*	• Best used when $a = 1$ and b is even	• May involve difficult calculations
Quadratic Formula *(Lesson 9.5)*	• Can be used for any quadratic equation • Gives exact solutions	• Takes time to do calculations

EXAMPLE 5 Choosing a Method

Solve the equation using any method. Explain your choice of method.

a. $x^2 - 10x = 1$ **b.** $2x^2 - 13x - 24 = 0$ **c.** $x^2 + 8x + 12 = 0$

a. The coefficient of the x^2-term is 1, and the coefficient of the x-term is an even number. So, solve by completing the square.

$x^2 - 10x = 1$	Write the equation.
$x^2 - 10x + 25 = 1 + 25$	Complete the square for $x^2 - 10x$.
$(x - 5)^2 = 26$	Write the left side as the square of a binomial.
$x - 5 = \pm\sqrt{26}$	Take the square root of each side.
$x = 5 \pm \sqrt{26}$	Add 5 to each side.

▶ So, the solutions are $x = 5 + \sqrt{26} \approx 10.1$ and $x = 5 - \sqrt{26} \approx -0.1$.

b. The equation is not easily factorable, and the numbers are somewhat large. So, solve using the Quadratic Formula.

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	Quadratic Formula
$= \dfrac{-(-13) \pm \sqrt{(-13)^2 - 4(2)(-24)}}{2(2)}$	Substitute 2 for a, −13 for b, and −24 for c.
$= \dfrac{13 \pm \sqrt{361}}{4}$	Simplify.
$= \dfrac{13 \pm 19}{4}$	Evaluate the square root.

▶ So, the solutions are $x = \dfrac{13 + 19}{4} = 8$ and $x = \dfrac{13 - 19}{4} = -\dfrac{3}{2}$.

c. The equation is easily factorable. So, solve by factoring.

$x^2 + 8x + 12 = 0$	Write the equation.
$(x + 2)(x + 6) = 0$	Factor the polynomial.
2 $x + 2 = 0$ or $x + 6 = 0$	Zero-Product Property
$x = -2$ or $x = -6$	Solve for x.

▶ The solutions are $x = -2$ and $x = -6$.

Solve the equation using any method. Explain your choice of method.

13. $x^2 + 11x - 12 = 0$

14. $9x^2 - 5 = 4$

15. $5x^2 - x - 1 = 0$

16. $x^2 = 2x - 5$

13. $x = -12$, $x = 1$; *Sample answer:* The equation is easily factorable, so solve by factoring.

14. $x = 1$, $x = -1$; *Sample answer:* The equation can be written in the form $x^2 = d$, so solve using square roots.

15. $x \approx 0.56$, $x \approx -0.36$; *Sample answer:* The equation is not factorable and the coefficient of the x^2-term is not 1, so solve using the quadratic formula.

16. no real solutions; *Sample answer:* The coefficient of the x^2-term is 1 and b is even, so solve by completing the square.