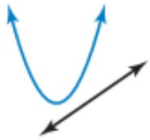
What You Will Learn

- Solve systems of nonlinear equations by graphing.
- Solve systems of nonlinear equations algebraically.
- Approximate solutions of nonlinear systems and equations.

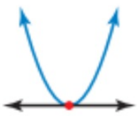
Solving Nonlinear Systems by Graphing

The methods for solving systems of linear equations can also be used to solve *systems of nonlinear equations*. A **system of nonlinear equations** is a system in which at least one of the equations is nonlinear.

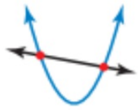
When a nonlinear system consists of a linear equation and a quadratic equation, the graphs can intersect in zero, one, or two points. So, the system can have zero, one, or two solutions, as shown.



No solutions



One solution



Two solutions

EXAMPLE 1 Solving a Nonlinear System by Graphing

Solve the system by graphing.

$y = 2x^2 + 5x - 1$ Equation 1

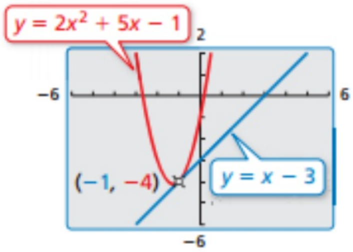
$y = x - 3$ Equation 2

SOLUTION

Step 1 Graph each equation.

Step 2 Estimate the point of intersection. The graphs appear to intersect at $(-1, -4)$.

Step 3 Check the point from Step 2 by substituting the coordinates into each of the original equations.



Equation 1

$$y = 2x^2 + 5x - 1$$
$$-4 \stackrel{?}{=} 2(-1)^2 + 5(-1) - 1$$
$$-4 = -4 \checkmark$$

Equation 2

$$y = x - 3$$
$$-4 \stackrel{?}{=} -1 - 3$$
$$-4 = -4 \checkmark$$

▶ The solution is $(-1, -4)$.

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Solve the system by graphing.

1. $y = x^2 + 4x - 4$
$y = 2x - 5$

2. $y = -x + 6$
$y = -2x^2 - x + 3$

3. $y = 3x - 15$
$y = \frac{1}{2}x^2 - 2x - 7$

MONITORING PROGRESS ANSWERS

1. $(-1, -7)$
2. no solutions
3. $(2, -9), (8, 9)$

EXAMPLE 2 Solving a Nonlinear System by Substitution

Solve the system by substitution.

$y = x^2 + x - 1$ Equation 1

$y = -2x + 3$ Equation 2

SOLUTION

Step 1 The equations are already solved for y.

Step 2 Substitute $-2x + 3$ for y in Equation 1 and solve for x.

$-2x + 3 = x^2 + x - 1$	Substitute $-2x + 3$ for y in Equation 1.
$3 = x^2 + 3x - 1$	Add $2x$ to each side.
$0 = x^2 + 3x - 4$	Subtract 3 from each side.
$0 = (x + 4)(x - 1)$	Factor the polynomial.
$x + 4 = 0$ or $x - 1 = 0$	Zero-Product Property
$x = -4$ or $x = 1$	Solve for x.

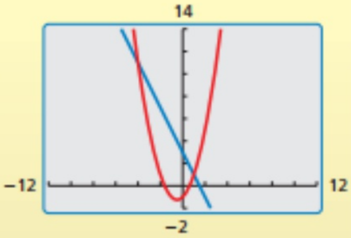
Step 3 Substitute -4 and 1 for x in Equation 2 and solve for y.

$y = -2(-4) + 3$	Substitute for x in Equation 2.	$y = -2(1) + 3$
$= 11$	Simplify.	$= 1$

▶ So, the solutions are $(-4, 11)$ and $(1, 1)$.

Check

Use a graphing calculator to check your answer. Notice that the graphs have two points of intersection at $(-4, 11)$ and $(1, 1)$.



EXAMPLE 3 Solving a Nonlinear System by Elimination

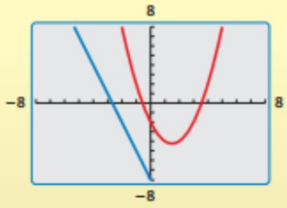
Solve the system by elimination.

$y = x^2 - 3x - 2$ Equation 1

$y = -3x - 8$ Equation 2

Check

Use a graphing calculator to check your answer. The graphs do not intersect.



SOLUTION

Step 1 Because the coefficients of the y-terms are the same, you do not need to multiply either equation by a constant.

Step 2 Subtract Equation 2 from Equation 1.

$$y = x^2 - 3x - 2 \qquad \text{Equation 1}$$
$$\underline{y = \quad -3x - 8} \qquad \text{Equation 2}$$
$$0 = x^2 \quad + 6 \qquad \text{Subtract the equations.}$$

Step 3 Solve for x.

$$0 = x^2 + 6 \qquad \text{Resulting equation from Step 2}$$
$$-6 = x^2 \qquad \text{Subtract 6 from each side.}$$

The square of a real number cannot be negative. So, the system has no real solutions.

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Solve the system by substitution.

4. $y = x^2 + 9$
$y = 9$

5. $y = -5x$
$y = x^2 - 3x - 3$

6. $y = -3x^2 + 2x + 1$
$y = 5 - 3x$

Solve the system by elimination.

7. $y = x^2 + x$
$y = x + 5$

8. $y = 9x^2 + 8x - 6$
$y = 5x - 4$

9. $y = 2x + 5$
$y = -3x^2 + x - 4$

Approximating Solutions

When you cannot find the exact solution(s) of a system of equations, you can analyze output values to approximate the solution(s).

MONITORING PROGRESS ANSWERS

4. (0, 9)
5. (−3, 15), (1, −5)
6. no solutions
7. about (2.24, 7.24), about (−2.24, 2.76)
8. $\left(\frac{1}{3}, -\frac{7}{3}\right), \left(-\frac{2}{3}, -\frac{22}{3}\right)$
9. no solutions

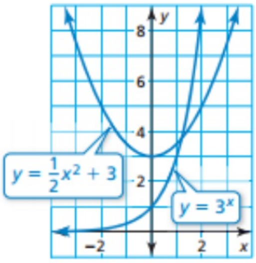
EXAMPLE 4 Approximating Solutions of a Nonlinear System

Approximate the solution(s) of the system to the nearest thousandth.

$y = \frac{1}{2}x^2 + 3$ Equation 1

$y = 3^x$ Equation 2

SOLUTION



Sketch a graph of the system. You can see that the system has one solution between $x = 1$ and $x = 2$.

Substitute 3^x for y in Equation 1 and rewrite the equation.

$3^x = \frac{1}{2}x^2 + 3$ — Substitute 3^x for y in Equation 1.

$3^x - \frac{1}{2}x^2 - 3 = 0$ — Rewrite the equation.

Because you do not know how to solve this equation algebraically, let $f(x) = 3^x - \frac{1}{2}x^2 - 3$. Then evaluate the function for x-values between 1 and 2.

$f(1.1) \approx -0.26$

$f(1.2) \approx 0.02$

Because $f(1.1) < 0$ and $f(1.2) > 0$, the zero is between 1.1 and 1.2.

REMEMBER

The function values that are closest to 0 correspond to x-values that best approximate the zeros of the function.

$f(1.2)$ is closer to 0 than $f(1.1)$, so decrease your guess and evaluate $f(1.19)$.

$f(1.19) \approx -0.012$	Because $f(1.19) < 0$ and $f(1.2) > 0$, the zero is between 1.19 and 1.2. So, increase guess.
$f(1.191) \approx -0.009$	Result is negative. Increase guess.
$f(1.192) \approx -0.006$	Result is negative. Increase guess.
$f(1.193) \approx -0.003$	Result is negative. Increase guess.
$f(1.194) \approx -0.0002$	Result is negative. Increase guess.
$f(1.195) \approx 0.003$	Result is positive.

Because $f(1.194)$ is closest to 0, $x \approx 1.194$.

Substitute $x = 1.194$ into one of the original equations and solve for y.

$y = \frac{1}{2}x^2 + 3 = \frac{1}{2}(1.194)^2 + 3 \approx 3.713$

So, the solution of the system is about (1.194, 3.713).