9.2 Solving Quadratic Equations by Graphing

- Solve quadratic equations by graphing.
- Use graphs to find and approximate the zeros of functions.
- Solve real-life problems using graphs of quadratic functions.

Solving Quadratic Equations by Graphing

A **quadratic equation** is a nonlinear equation that can be written in the standard form $ax^2 + bx + c = 0$, where $a \neq 0$.

In Chapter 7, you solved quadratic equations by factoring. You can also solve quadratic equations by graphing.

Solving Quadratic Equations by Graphing

Step 1 Write the equation in standard form, $ax^2 + bx + c = 0$.

Step 2 Graph the related function $y = ax^2 + bx + c$.

Step 3 Find the x-intercepts, if any.

The solutions, or *roots*, of $ax^2 + bx + c = 0$ are the x-intercepts of the graph.

EXAMPLE 1 Solving a Quadratic Equation: Two Real Solutions

Solve $x^2 + 2x = 3$ by graphing.

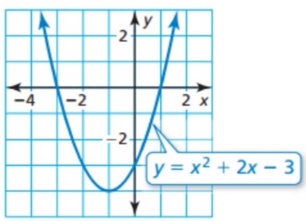
SOLUTION

Step 1 Write the equation in standard form.

$x^2 + 2x = 3$ Write original equation.

$x^2 + 2x - 3 = 0$ Subtract 3 from each side.

Step 2 Graph the related function $y = x^2 + 2x - 3$.



Step 3 Find the x-intercepts. The x-intercepts are -3 and 1.

▶ So, the solutions are $x = -3$ and $x = 1$.

Check

$x^2 + 2x = 3$	Original equation	$x^2 + 2x = 3$
$(-3)^2 + 2(-3) \stackrel{?}{=} 3$	Substitute.	$1^2 + 2(1) \stackrel{?}{=} 3$
$3 = 3$ ✓	Simplify.	$3 = 3$ ✓

Solve the equation by graphing. Check your solutions.

1. $x^2 - x - 2 = 0$
2. $x^2 + 7x = -10$
3. $x^2 + x = 12$

1. $x = 2, x = -1$
2. $x = -5, x = -2$
3. $x = -4, x = 3$

EXAMPLE 2 Solving a Quadratic Equation: One Real Solution

Solve $x^2 - 8x = -16$ by graphing.

Solve $x^2 - 8x = -16$ by graphing.

SOLUTION

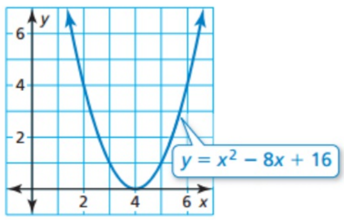
Step 1 Write the equation in standard form.

$$x^2 - 8x = -16 \quad \text{Write original equation.}$$

$$x^2 - 8x + 16 = 0 \quad \text{Add 16 to each side.}$$

Step 2 Graph the related function $y = x^2 - 8x + 16$.

Step 3 Find the x-intercept. The only x-intercept is at the vertex, (4, 0).



▶ So, the solution is $x = 4$.

ANOTHER WAY

You can also solve the equation in Example 2 by factoring.

$$x^2 - 8x + 16 = 0$$

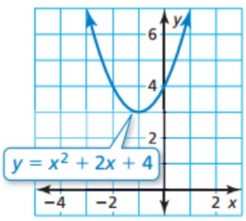
$$(x - 4)(x - 4) = 0$$

So, $x = 4$.

EXAMPLE 3 **Solving a Quadratic Equation: No Real Solutions**

Solve $-x^2 = 2x + 4$ by graphing.

SOLUTION



Method 1 Write the equation in standard form, $x^2 + 2x + 4 = 0$. Then graph the related function $y = x^2 + 2x + 4$, as shown at the left.

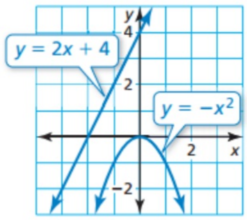
▶ There are no x-intercepts. So, $-x^2 = 2x + 4$ has no real solutions.

Method 2 Graph each side of the equation.

$y = -x^2$ Left side

$y = 2x + 4$ Right side

▶ The graphs do not intersect.
So, $-x^2 = 2x + 4$ has no real solutions.



Solve the equation by graphing.

4. $x^2 + 36 = 12x$

5. $x^2 + 4x = 0$

6. $x^2 + 10x = -25$

7. $x^2 = 3x - 3$

8. $x^2 + 7x = -6$

9. $2x + 5 = -x^2$

Number of Solutions of a Quadratic Equation

A quadratic equation has:

- two real solutions when the graph of its related function has two x-intercepts.
- one real solution when the graph of its related function has one x-intercept.
- no real solutions when the graph of its related function has no x-intercepts.

EXAMPLE 4 **Finding the Zeros of a Function**

The graph of $f(x) = (x - 3)(x^2 - x - 2)$ is shown. Find the zeros of f.

The x-intercepts are -1, 2, and 3.

▶ So, the zeros of f are -1, 2, and 3.

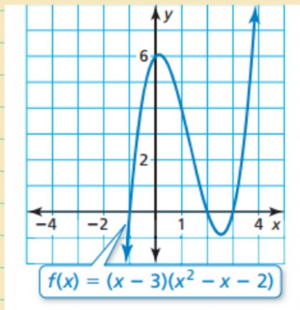
Check

$f(-1) = (-1 - 3)[(-1)^2 - (-1) - 2] = 0$ ✓

$f(2) = (2 - 3)(2^2 - 2 - 2) = 0$ ✓

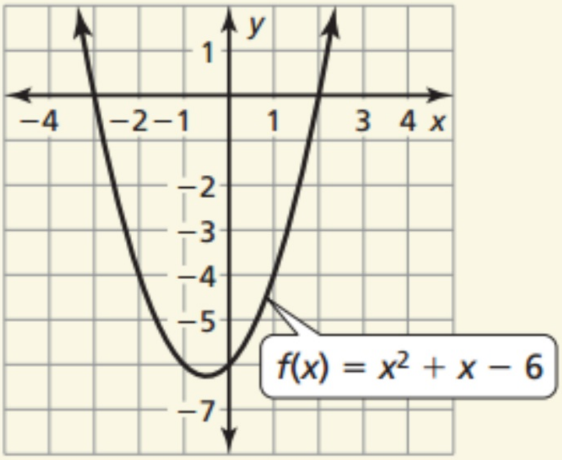
$f(3) = (3 - 3)(3^2 - 3 - 2) = 0$ ✓

The zeros of a function are not necessarily integers. To approximate zeros, analyze the signs of function values. When two function values have different signs, a zero lies between the x-values that correspond to the function values.



10. Graph $f(x) = x^2 + x - 6$. Find the zeros of f.

10.



−3, 2

EXAMPLE 6 **Real-Life Application**

A football player kicks a football 2 feet above the ground with an initial vertical velocity of 75 feet per second. The function $h = -16t^2 + 75t + 2$ represents the height h (in feet) of the football after t seconds. (a) Find the height of the football each second after it is kicked. (b) Use the results of part (a) to estimate when the height of the football is 50 feet. (c) Using a graph, after how many seconds is the football 50 feet above the ground?

Seconds, t	Height, h
0	2
1	61
2	88
3	83
4	46
5	−23

a. Make a table of values starting with $t = 0$ seconds using an increment of 1. Continue the table until a function value is negative.

▶ The height of the football is 61 feet after 1 second, 88 feet after 2 seconds, 83 feet after 3 seconds, and 46 feet after 4 seconds.

b. From part (a), you can estimate that the height of the football is 50 feet between 0 and 1 second and between 3 and 4 seconds.

▶ Based on the function values, it is reasonable to estimate that the height of the football is 50 feet slightly less than 1 second and slightly less than 4 seconds after it is kicked.

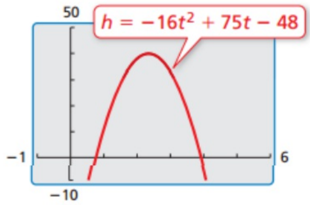
c. To determine when the football is 50 feet above the ground, find the t-values for which $h = 50$. So, solve the equation $-16t^2 + 75t + 2 = 50$ by graphing.

Step 1 Write the equation in standard form.

$-16t^2 + 75t + 2 = 50$ Write the equation.

$-16t^2 + 75t - 48 = 0$ Subtract 50 from each side.

Step 2 Use a graphing calculator to graph the related function $h = -16t^2 + 75t - 48$.



Step 3 Use the *zero* feature to find the zeros of the function.